

Recurrence Relation Problems

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| 1. $a_n - 2a_{n-1} = 0;$ | $a_0 = 3.$ |
| 2. $a_n = 6a_{n-1} + 3;$ | $a_0 = -8/5.$ |
| 3. $a_n + 3a_{n-1} = n + 2;$ | $a_0 = 37/16.$ |
| 4. $2a_n - a_{n-1} - 2^n = 0;$ | $a_0 = 1.$ |
| 5. $3a_n = -2a_{n-1} + 3n4^n;$ | $a_0 = 2.$ |
| 6. $a_n - 6n - 10 = a_{n-1};$ | $a_0 = -3.$ |
| 7. $a_n = 3a_{n-1} + 2 \cdot 3^n;$ | $a_0 = 10.$ |
| 8. $a_n - 2a_{n-1} = -4n2^n;$ | $a_0 = 5.$ |
| 9. $a_n + 4a_{n-1} = 10n^2 + 1;$ | $a_0 = 1.$ |
| 10. $a_n + a_{n-1} - 4n(-1)^n = 0;$ | $a_0 = -3.$ |

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| 11. $a_n - 5a_{n-1} + 6a_{n-2} = 0;$ | $a_0 = 5;$ | $a_1 = 12.$ |
| 12. $a_n = 5a_{n-1} - 4a_{n-2} + 28n;$ | $a_0 = 0;$ | $a_1 = -136/7.$ |
| 13. $a_n + a_{n-1} = 6a_{n-2};$ | $a_0 = 10;$ | $a_1 = 0.$ |
| 14. $a_n - a_{n-2} + 2^n;$ | $a_0 = 1;$ | $a_1 = 1.$ |
| 15. $a_n - 72n^2 + 5a_{n-1} + 6a_{n-2} = 0;$ | $a_0 = 175/12;$ | $a_1 = 451/12.$ |
| 16. $2a_n + 3a_{n-2} = 7a_{n-1} + 9n2^n;$ | $a_0 = -32;$ | $a_1 = -181/2.$ |
| 17. $6a_n + a_{n-1} = 2a_{n-2} + 5n + 13;$ | $a_0 = 3;$ | $a_1 = 35/6.$ |
| 18. $a_n + 4a_{n-1} - 45 + 4a_{n-2} = 0;$ | $a_0 = 10;$ | $a_1 = -3.$ |
| 19. $a_n + 9a_{n-2} = 6a_{n-1} + 8;$ | $a_0 = 2;$ | $a_1 = 20.$ |
| 20. $a_n = -a_{n-2};$ | $a_0 = 2;$ | $a_1 = 0.$ |
| 21. $a_n = 2a_{n-1} + a_{n-2} + 4(-1)^n;$ | $a_0 = 4;$ | $a_1 = -4.$ |
| 22. $a_n + 2a_{n-2} = 2a_{n-1} + 25n3^n;$ | $a_0 = 22;$ | $a_1 = 67.$ |
| 23. $a_n - 8a_{n-1} + 15a_{n-2} = 12 \cdot 3^n;$ | $a_0 = 0;$ | $a_1 = -8.$ |
| 24. $a_n + 5 + 2a_{n-2} = 3a_{n-1} + 2n;$ | $a_0 = -1;$ | $a_1 = 2.$ |
| 25. $a_n - 2a_{n-1} + a_{n-2} = 6n - 10;$ | $a_0 = 2;$ | $a_1 = 2.$ |
| 26. $a_n = 4a_{n-1} - 4a_{n-2} + 6 \cdot 2^n;$ | $a_0 = 5;$ | $a_1 = 16.$ |

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| 27. $a_n - 3a_{n-1} + 3a_{n-2} - a_{n-3} = 88n - 30;$ | $a_0 = 1;$ | $a_1 = 3;$ | $a_2 = 25.$ |
| 28. $a_n - a_{n-4} = 0;$ | $a_0 = 4;$ | $a_1 = 2;$ | $a_2 = 4; a_3 = 2.$ |
| 29. $a_n - 5a_{n-1} + 8a_{n-2} - 4a_{n-3} = 2^n$ | $a_0 = 0;$ | $a_1 = 0;$ | $a_2 = 8.$ |
| 30. $a_n - 2a_{n-2} + a_{n-4} = 0;$ | $a_0 = 2;$ | $a_1 = 0;$ | $a_2 = 6; a_3 = 0.$ |

Homogeneous Problem

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Forcing Function

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| 1. $a_n - 2a_{n-1} = 0;$ | 0. |
| 2. $a_n - 6a_{n-1} = 0;$ | 3. |
| 3. $a_n + 3a_{n-1} = 0;$ | $n + 2.$ |
| 4. $2a_n - a_{n-1} = 0;$ | $2^n.$ |
| 5. $3a_n + 2a_{n-1} = 0;$ | $3n4^n.$ |
| 6. $a_n - a_{n-1} = 0;$ | $6n + 10.$ |
| 7. $a_n - 3a_{n-1} = 0;$ | $2 \cdot 3^n.$ |
| 8. $a_n - 2a_{n-1} = 0;$ | $-4n2^n.$ |
| 9. $a_n + 4a_{n-1} = 0;$ | $10n^2 + 1.$ |
| 10. $a_n + a_{n-1} = 0;$ | $4n(-1)^n.$ |

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| 11. $a_n - 5a_{n-1} + 6a_{n-2} = 0;$ | 0. |
| 12. $a_n - 5a_{n-1} + 4a_{n-2} = 0;$ | $28n.$ |
| 13. $a_n + a_{n-1} - 6a_{n-2} = 0;$ | 0. |
| 14. $a_n - a_{n-2} = 0;$ | $2^n.$ |
| 15. $a_n + 5a_{n-1} + 6a_{n-2} = 0;$ | $72n^2.$ |
| 16. $2a_n - 7a_{n-1} + 3a_{n-2} = 0;$ | $9n2^n.$ |
| 17. $6a_n + a_{n-1} - 2a_{n-2} = 0;$ | $5n + 13$ |
| 18. $a_n + 4a_{n-1} + 4a_{n-2} = 0;$ | 45. |
| 19. $a_n - 6a_{n-1} + 9a_{n-2} = 0;$ | 8. |
| 20. $a_n + a_{n-2} = 0;$ | 0. |
| 21. $a_n - 2a_{n-1} - a_{n-2} = 0;$ | $4(-1)^n.$ |
| 22. $a_n - 2a_{n-1} + 2a_{n-2} = 0;$ | $25n3^n.$ |
| 23. $a_n - 8a_{n-1} + 15a_{n-2} = 0;$ | $12 \cdot 3^n.$ |
| 24. $a_n - 3a_{n-1} + 2a_{n-2} = 0;$ | $2n - 5.$ |
| 25. $a_n - 2a_{n-1} + a_{n-2} = 0;$ | $6n - 10.$ |
| 26. $a_n - 4a_{n-1} + 4a_{n-2} = 0;$ | $6 \cdot 2^n.$ |

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| 27. $a_n - 3a_{n-1} + 3a_{n-2} - a_{n-3} = 0;$ | $88n - 30.$ |
| 28. $a_n - a_{n-4} = 0;$ | 0. |
| 29. $a_n - 5a_{n-1} + 8a_{n-2} - 4a_{n-3} = 0;$ | $2^n.$ |
| 30. $a_n - 2a_{n-2} + a_{n-4} = 0;$ | 0. |

Characteristic Poly	Roots	General Homo Solution
1. $x - 2 = 0$;	2	$a_n = A2^n$.
2. $x - 6 = 0$;	6	$a_n = A6^n$.
3. $x + 3 = 0$;	-3	$a_n = A(-3)^n$.
4. $2x - 1 = 0$;	1/2	$a_n = A(1/2)^n$.
5. $3x + 2 = 0$;	-2/3	$a_n = A(-2/3)^n$.
6. $x - 1 = 0$;	1	$a_n = A$.
7. $x - 3 = 0$;	3	$a_n = A3^n$.
8. $x - 2 = 0$;	2	$a_n = A2^n$.
9. $x + 4 = 0$;	-4	$a_n = A(-4)^n$.
10. $x + 1 = 0$;	-1	$a_n = A(-1)^n$.
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11. $x^2 - 5x + 6 = 0$;	2, 3	$a_n = A2^n + B3^n$.
12. $x^2 - 5x + 4 = 0$;	1, 4	$a_n = A + B4^n$.
13. $x^2 + x - 6 = 0$;	2, -3	$a_n = A2^n + B(-3)^n$.
14. $x^2 - 1 = 0$;	1, -1	$a_n = A + B(-1)^n$.
15. $x^2 + 5x + 6 = 0$;	-2, -3	$a_n = A(-2)^n + B(-3)^n$.
16. $2x^2 - 7x + 3 = 0$;	1/2, 3	$a_n = A(1/2)^n + B3^n$.
17. $6x^2 + x - 2 = 0$;	1/2, -2/3	$a_n = A(1/2)^n + B(-2/3)^n$.
18. $x^2 + 4x + 4 = 0$;	-2, -2	$a_n = A(-2)^n + Bn(-2)^n$.
19. $x^2 - 6x + 9 = 0$;	3, 3	$a_n = A3^n + Bn3^n$.
20. $x^2 + 1 = 0$;	i, -i	$a_n = Ai^n + B(-i)^n$.
21. $x^2 - 2x - 1 = 0$;	$1 \pm \sqrt{2}$	$a_n = A(1+\sqrt{2})^n + B(1-\sqrt{2})^n$.
22. $x^2 - 2x + 2 = 0$;	$1 \pm i$	$a_n = A(1+i)^n + B(1-i)^n$.
23. $x^2 - 8x + 15 = 0$;	3, 5	$a_n = A3^n + B5^n$.
24. $x^2 - 3x + 2 = 0$;	1, 2	$a_n = A + B2^n$.
25. $x^2 - 2x + 1 = 0$;	1, 1	$a_n = A + Bn$.
26. $x^2 - 4x + 4 = 0$;	2, 2	$a_n = A2^n + Bn2^n$.
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27. $x^3 - 3x^2 + 3x - 1 = 0$;	1, 1, 1	$a_n = A + Bn + Cn^2$.
28. $x^4 - 1 = 0$;	$\pm 1, \pm i$	$a_n = A + B(-1)^n + Ci^n + D(-i)^n$.
29. $x^3 - 5x^2 + 8x - 4 = 0$;	1, 2, 2	$a_n = A + B2^n + Cn2^n$.
30. $x^4 - 2x^2 + 1 = 0$;	$\pm 1, \pm i$	$a_n = A + B(-1)^n + Cn + Dn(-1)^n$.

Particular Solution--Guess and Answer

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| 1. $a_n = 0$; (it's homo) | $a_n = 0$. |
| 2. $a_n = A$; | $a_n = -3/5$. |
| 3. $a_n = An + B$; | $a_n = (1/4)n + 5/16$. |
| 4. $a_n = A2^n$; | $a_n = (2/3)2^n$. |
| 5. $a_n = An4^n + B4^n$; | $a_n = (6/7)n4^n + (6/49)4^n$. |
| 6. $a_n = n(An + B)$; | $a_n = 3n^2 - 7n$. |
| 7. $a_n = An3^n$; | $a_n = 2n3^n$. |
| 8. $a_n = n(An2^n + B2^n)$; | $a_n = -2n^22^n + 2n2^n$. |
| 9. $a_n = An^2 + Bn + C$; | $a_n = 2n^2 + (16/5)n - 19/25$. |
| 10. $a_n = n(An(-1)^n + B(-1)^n)$; | $a_n = 2n^2(-1)^n + 2n(-1)^n$. |
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| 11. $a_n = 0$; (it's homo) | $a_n = 0$. |
| 12. $a_n = n(An + B)$; | $a_n = -2n^2 - (38/7)n$. |
| 13. $a_n = 0$; (it's homo) | $a_n = 0$. |
| 14. $a_n = A2^n$; | $a_n = (4/3)2^n$. |
| 15. $a_n = An^2 + Bn + C$; | $a_n = 6n^2 + 17n + 175/12$. |
| 16. $a_n = An2^n + B2^n$; | $a_n = -12n2^n - 32 \cdot 2^n$. |
| 17. $a_n = An + B$; | $a_n = n + 2$. |
| 18. $a_n = A$; | $a_n = 5$. |
| 19. $a_n = A$; | $a_n = 2$. |
| 20. $a_n = 0$; (it's homo) | $a_n = 0$. |
| 21. $a_n = A(-1)^n$; | $a_n = 2(-1)^n$. |
| 22. $a_n = An3^n + B3^n$; | $a_n = 45n3^n - 18 \cdot 3^n$. |
| 23. $a_n = n(A3^n)$; | $a_n = -2n3^n$. |
| 24. $a_n = n(An + B)$; | $a_n = -n^2$. |
| 25. $a_n = n^2(An + B)$; | $a_n = n^3 - 2n^2$. |
| 26. $a_n = An^22^n$; | $a_n = 3n^22^n$. |
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| 27. $a_n = n^2(An + B)$; | $a_n = n^4 + n^3$. |
| 28. $a_n = 0$; (it's homo) | $a_n = 0$. |
| 29. $a_n = An^22^n$; | $a_n = n^22^n$. |
| 30. $a_n = 0$; (it's homo) | $a_n = 0$. |

General Solution

1. $a_n = A2^n.$
2. $a_n = A6^n - 3/5.$
3. $a_n = A(-3)^n + (1/4)n + 5/16.$
4. $a_n = A(1/2)^n + (2/3)2^n.$
5. $a_n = A(-2/3)^n + (6/7)n4^n + (6/49)4^n.$
6. $a_n = A + 3n^2 - 7n.$
7. $a_n = A3^n + 2n3^n.$
8. $a_n = A2^n - 2n^22^n + 2n2^n.$
9. $a_n = A(-4)^n + 2n^2 + (16/5)n - 19/25.$
10. $a_n = A(-1)^n + 2n^2(-1)^n + 2n(-1)^n.$

Initial Value Solutions

- A = 3.
 A = -1.
 A = 2.
 A = 1/3.
 A = 92/49.
 A = -3.
 A = 10.
 A = 5.
 A = 31/25.
 A = -3.

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| 11. $a_n = A2^n + B3^n.$ | A = 3; | B = 2. |
| 12. $a_n = A + B4^n - 2n^2 - (38/7)n.$ | A = 4; | B = -4. |
| 13. $a_n = A2^n + B(-3)^n.$ | A = 6; | B = 4. |
| 14. $a_n = A + B(-1)^n + (4/3)2^n.$ | A = -1; | B = 2/3. |
| 15. $a_n = A(-2)^n + B(-3)^n + 6n^2 + 17n + 175/12.$ | A = 0; | B = 0. |
| 16. $a_n = A(1/2)^n + B3^n - 12n2^n - 32 \cdot 2^n.$ | A = 1; | B = -1. |
| 17. $a_n = A(1/2)^n + B(-2/3)^n + n + 2.$ | A = 3; | B = -2. |
| 18. $a_n = A(-2)^n + Bn(-2)^n + 5.$ | A = 5; | B = -1. |
| 19. $a_n = A3^n + Bn3^n + 2.$ | A = 0; | B = 6. |
| 20. $a_n = Ai^n + B(-i)^n.$ | A = 1; | B = 1. |
| 21. $a_n = A(1+\sqrt{2})^n + B(1-\sqrt{2})^n + 2(-1)^n.$ | A = $1-\sqrt{2}$; | B = $1+\sqrt{2}$. |
| 22. $a_n = A(1+i)^n + B(1-i)^n + 45n3^n - 18 \cdot 3^n.$ | A = 2; | B = 2; |
| 23. $a_n = A3^n + B5^n - 2n3^n.$ | A = 1; | B = -5. |
| 24. $a_n = A + B2^n - n^2.$ | A = -2; | B = 1. |
| 25. $a_n = A + Bn + n^3 - 2n^2.$ | A = 2; | B = 1. |
| 26. $a_n = A2^n + Bn2^n + 3n^22^n.$ | A = 5; | B = 0. |

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| 27. $a_n = A + Bn + Cn^2 + n^4 + n^3.$ | A = 1; | B = 0; | C = 0. |
| 28. $a_n = A + B(-1)^n + Ci^n + D(-i)^n.$ | A = 3; | B = 1; | C = 0; D = 0. |
| 29. $a_n = A + B2^n + Cn2^n + n^22^n.$ | A = 0; | B = 0; | C = -1. |
| 30. $a_n = A + B(-1)^n + Cn + Dn(-1)^n.$ | A = 1; | B = 1; | C = 1; D = 1. |