MAT 313 — Test I

I (a) For what values of p does $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converge? diverge? ____, ____.

(10) (b) For what values of r does $\sum_{n=1}^{\infty} r^n$ converge? diverge? ____, ____.

Your answers to I may be useful in what follows.

II True or false?

(a) $\sum \frac{2}{2^n+3}$ is convergent. ______.

(b) $\sum \frac{n^2}{15n^2+30n+1}$ is divergent. ______.

(c) $\sum \frac{n^2}{2^n}$ is convergent ______.

(35) (d) If the nth term of a series approaches zero with n, then that series will converge ______.

(e) If $\sum a_n$ and $\sum b_n$ are series such that $\lim \frac{a_n}{b_n} = 1$, then $\sum a_n$ converges $\Leftrightarrow$ $\sum b_n$ converges. ______.

(f) The integral test is useful in showing that the series $\sum \frac{(-1)^n}{n}$ is convergent. ______.

(g) $\sum \frac{(-1)^n}{n^2}$ is conditionally convergent. ______.

III Use the integral test to determine convergence or divergence

(15) of $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^2}$.

IV Show that $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \log n}$ converges conditionally.

V Does $\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{9} + \frac{1}{8} - \frac{1}{27} + \frac{1}{16} - \cdots$ converge or diverge? Justify your answer.