MATH 32

1. PARALLELGRAM LAW: FOR ANY TWO VECTORS $\vec{A}$ AND $\vec{B}$ THEN: $|\vec{A}+\vec{B}|^2 + |\vec{A}-\vec{B}|^2 = 2|\vec{A}|^2 + 2|\vec{B}|^2$

   (HINT: USE DOT PRODUCT)

2. LET a_1, a_2, a_3 & b_1, b_2, b_3 BE REAL NUMBERS SHOW

$$(a_1b_1 + a_2b_2 + a_3b_3)^2 \le (a_1^2+a_2^2+a_3^2)(b_1^2+b_2^2+b_3^2)$$

   (HINT: DOT PRODUCT)

3. $\vec{A}$ AND $\vec{B}$ ARE UNIT VECTORS AND $\vec{A}\cdot\vec{B} = 0$ THEN

$$\vec{A}\times\vec{B} = \vec{A}\times(\vec{A}\times(\vec{A}\times(\vec{A}\times\vec{B})))$$

4. THE CURVE $\vec{R}(s) = \langle s, f(s), g(s)\rangle$ IS ALREADY PARAMETERIZED BY ARCLENGTH; WHAT MUST $f(s)$ AND $g(s)$ BE?

5. FIND $\vec{B}$ THE BI-NORMAL TO THE CURVE $\vec{R}(t) = \langle t, \sin t, \sin t\rangle$ USE $\vec{B}$ TO SHOW THAT THE TORSION, $\tau = 0$. SO $\vec{R}(t)$ ALWAYS LIES IN A PLANE, WHAT IS THAT PLANE?

6. GIVEN TWO LINES $\ell_1: \vec{x} = \vec{x}_1 + t\vec{D}_1$ $\ell_2: \vec{x} = \vec{x}_2 + t\vec{D}_2$ AND LET m BE A LINE CONTAINING A COMMON PERPENDICULAR TO ℓ_1 AND ℓ_2. FIND AN EQUATION OF A PLANE THROUGH (CONTAINING) TWO OF THE LINES ℓ_1, ℓ_2, m.

7. EXPRESS THE EQUATION

$$f(x,y) = \frac{x^2-y^2}{2xy}$$

   (BOTH x AND y NOT ZERO)
   (ie NOT DEFINED ON x or y-a

   IN BOTH CYLINDER AND SPHERICAL CO-ORDINATES

8. TWO PLANES $\vec{N}_1\cdot(\vec{x}-\vec{x}_1) = 0$ AND $\vec{N}_2\cdot(\vec{x}-\vec{x}_2)$ INTERSECT IN A LINE WHICH CONTAINS THE POINT $\vec{x}_0$. WHAT IS ~~THE~~ AN EQUATION OF THE LINE?

9. FIND FORMULAS FOR THE ANGLE BETWEEN A LINE AND A PLANE BETWEEN TWO INTERSECTING PLANES (SEE PROB'S 14, 15 P. 530)

10. THE EQUATION $\left\{\begin{array}{l} x = a\cos t \\ y = a\sin t \\ z = bt \end{array}\right\}$ ARE OF A CIRCULAR HELIX

    TRANSLATE THESE EQUATIONS INTO BOTH CYLINDRICAL AND SPHERICAL CO-ORDINATES

TEST #1 MATH 32

WRITE ON ONE SIDE OF PAPER; NEATNESS AND READABLITY WILL COUNT

SHOW ALL WORK

1. a) LET $A(1,2,3)$, $B(-1,2,1)$ AND $C(4,3,2)$ BE POINTS.
   FIND THE DISTANCE FROM THE POINT A TO THE LINE THROUGH THE POINTS B AND C.

   b) FIND THE DISTANCE FROM THE ORIGIN TO THE PLANE THROUGH THE POINTS A, B AND C.

2. LET $\vec{X} = \langle x, y, z \rangle$, $\vec{X}_1 = \langle 2, 3, -4 \rangle$, $\vec{D}_1 = \langle 3, 4, 2 \rangle$, $\vec{N} = \langle 4, 5, 6 \rangle$, $\vec{Y}_1 = \langle 3, 9, 5 \rangle$. FIND THE CO-ORDINATES OF THE POINT P IN WHICH THE LINE $\vec{X} = \vec{X}_1 + t\vec{D}_1$ INTERSECTS THE PLANE $\vec{N} \cdot (\vec{X} - \vec{Y}_1) = 0$

3. LET $\overrightarrow{R(t)}$ BE A CURVE; PROVE:

$$\overrightarrow{R(t)} \cdot \frac{d\overrightarrow{R(t)}}{dt} \equiv 0 \quad \text{IF AND ONLY IF} \quad |\overrightarrow{R(t)}| \text{ IS A CONSTANT.}$$

4. SHOW THAT THE INTERSECTION OF THE HYPERBOLIC PARABOLOID $z = x^2 - y^2$ AND THE PLANE $x - y + 1 = 0$ IS THE LINE:

$$\begin{cases} x = t \\ y = \underline{1} + t \\ z = -\underline{1} - 2t \end{cases}$$

(IE. show if (x,y,z) lies on both surfaces, it lies on the line; and conversely, if (x,y,z) lies on the line, it lies on both surfaces.)

5. LET $\overrightarrow{R(t)} = \langle t, \sin t, \sin t \rangle$ FIND THE VELOCITY $\overrightarrow{V(t)}$, THE ACCELERATION $\overrightarrow{A(t)}$, THE BINORMAL $\overrightarrow{B(t)}$ (WHEN DEFINED), AND SHOW THAT IF $\overrightarrow{B(t)}$ IS DEFINED AT $t = a$ THEN THE TORSION $\tau = \left| \left. \frac{dB}{ds} \right|_{t=a} \right| = 0$.

YOU MAY USE THE FORMULA: $\overrightarrow{B(t)} = \begin{cases} \dfrac{\overrightarrow{V(t)} \times \overrightarrow{A(t)}}{|V(t) \times A(t)|} & \text{if } |V \times A| \neq 0 \\ \text{undefined} & \text{if } |V \times A| = 0 \end{cases}$

MATH 32 TEST 2

WRITE ON ONE SIDE OF PAPER; NEATNESS AND READABILITY WILL COUNT; SHOW ALL WORK

1. FIND $\frac{\partial z}{\partial s}$ AND $\frac{\partial z}{\partial t}$ WHERE $z = xe^y + ye^x$, $x = s\ln t$ and $y = t\ln s$.

2. FIND $\partial z/\partial x$ and $\partial z/\partial y$ BY IMPLICIT DIFFERENTIATION OF $xe^{yz} + ye^{xz} - y^2 + 3 = 0$.

3. LET $f(x,y,z) = x\ln x - ye^y + z^2$; FIND:

a) The gradient $\vec{\nabla f}$.

b) The directional derivative of f at the point $(1,1,1)$ in the direction $\langle\cos\alpha, \cos\beta, \cos\gamma\rangle$

c) Suppose $f(x,y,z)$ is the temperature at the point (x,y,z). You are at the pt. $(2,-1,3)$ and you want to move in a direction that cools you the fastest. Which way do you go?

d) Give an equation of the tangent plane to the level surface of f through the point $(e, \ln 2, 0)$

4. FIND THE MINIMUM AND MAXIMUM VALUES OF $g(x,y) = x^2 - y^2$ IN $x^2 + y^2 \le 1$

TEST 1B MATH 32

WRITE ON ONE SIDE OF PAPER; NEATNESS AND READABLITY WILL COUNT; SHOW ALL WORK.

1 a) Define $\vec{A} \times \vec{B}$; when can $\vec{A} \times \vec{B} = 0$ (LIST ALL CASES)

b) Define $\vec{A} \cdot \vec{B}$; When is $\vec{A} \cdot \vec{B} = 0$ (ALL CASES)

c) Explain why $\vec{A} \times \vec{B} = 0$ and $\vec{A} \cdot \vec{B} = 0$ implies $\vec{A} = 0$ or $\vec{B} = 0$

d) Gives examples to show that it is possible for $\vec{A} \times \vec{B} = 0$ but $\vec{A} \cdot \vec{B} \neq 0$ and $\vec{A} \cdot \vec{B} = 0$ but $\vec{A} \times \vec{B} \neq 0$

2. FOR THE CONICAL HELIX: $\overline{R(t)} = \langle e^t \cos t, e^t \sin t, e^t \rangle$ LET THE PARAMETER s BE ARCLENGTH. FIND THE TANGENT $\overline{T}(t)$; $\frac{ds}{dt}$; and use $\frac{ds}{dt}$ to find s as a function of t. Show that $\overline{R(t)}$ is well named, that is it lies on the cone $x^2 + y^2 - z^2 = 0$.

3. a) SHOW that the equation $\rho = \cos\varphi$ IS THE EQUATION of a sphere of radius $\frac{1}{2}$ and center $(0, 0, \frac{1}{2})$ by changing the equation into rectangle co-ordinates. [HINT: multiply by ρ]. What is the equation in cylinder co-ordinates?

b) Express $x^2 + y^2 - z^2 = 0$ in cylinder co-ordinates; in spherical co-ordinates. Which of these equations is it easiest to determine that the surface is a cone?

4. Prove: if $\overrightarrow{R(t)}$ is a curve then:

$\overline{R(t)}$ is always perpendicular to $\overrightarrow{V(t)}$ (the velocity) if and only if $|\overrightarrow{R(t)}|$ is a constant.

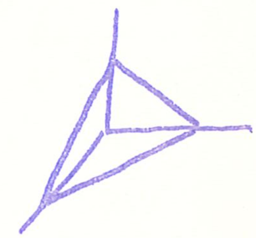
MATH 32 TEST 3

TAKE HOME DUE 8:00 WED MAY 23.

YOU MAY USE THE TEXT AND YOUR NOTES.

WRITE ON <u>ONE</u> SIDE OF EACH PAPER, SHOW <u>ALL</u> WORK, NEATNESS AND READABILITY WILL COUNT.

10 I. DETERMINE THE AREA OF THE TRIANGLE CUT FROM THE PLANE $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ BY THE CO-ORDINATE PLANES.



II. A) FIND THE CENTROID OF THE SOLID BOUNDED

10 BY THE CYLINDER $r = a$, THE CONE $z = r$, AND THE PLANE $z = 0$.

10 B) FIND I_z FOR THE SOLID IN PART A.

III. A) FIND THE VOLUME OF THE SOLID BOUNDED ABOVE BY THE SPHERE $\rho = a$ AND BELOW

10 BY THE CONE $\cos\varphi = k$ (REMEMBER THAT $(0 \le \varphi \le \pi)$).

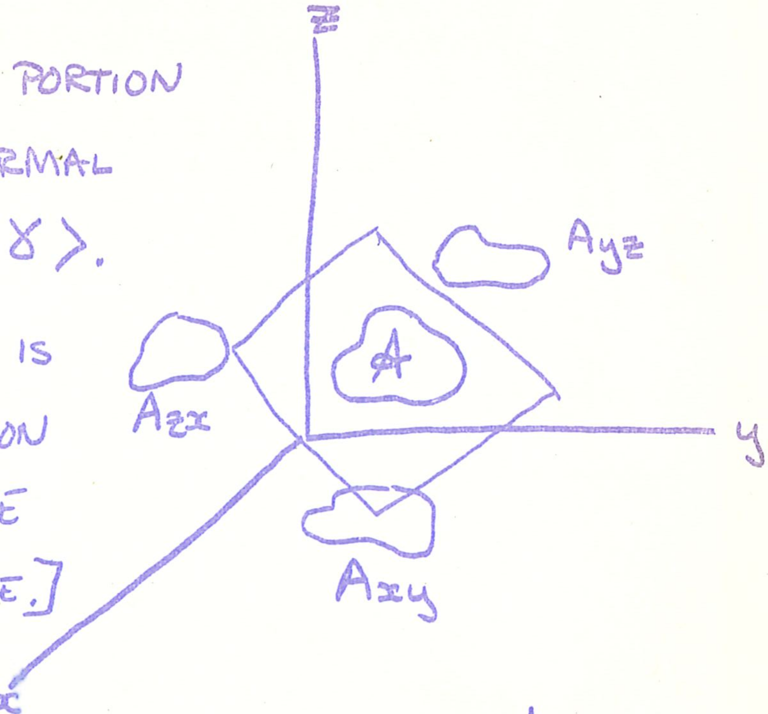
B) CHECK YOUR ANSWER BY FINDING THE

10 VOLUME OF THE SPHERE $\rho = a$ BY LETTING $\varphi = \pi$.

IV. FIND $\int_0^1 \int_0^2 \int_0^{2-x} z^4 \, dz \, dx \, dy$ BY FIRST

10 CHANGING THE ORDER OF INTEGRATION TO $dy \, dx \, dz$ AND THEN EVALUATING THE RESULTING INTEGRAL.

V A) Pythagorean Theorem for Areas:



$\mathcal{A}$ IS THE AREA OF A PORTION OF A PLANE WITH NORMAL $\vec{N} = \langle \cos\alpha, \cos\beta, \cos\gamma \rangle$.

A_{xy} [resp: A_{yz}, A_{zx}] IS THE AREA OF THE PROJECTION

~~10~~ .15 OF $\mathcal{A}$ INTO THE xy PLANE [resp: yz plane, zx PLANE.]

SHOW THAT:

$$\mathcal{A} = \left[A_{xy}^2 + A_{yz}^2 + A_{zx}^2\right]^{\frac{1}{2}}.$$

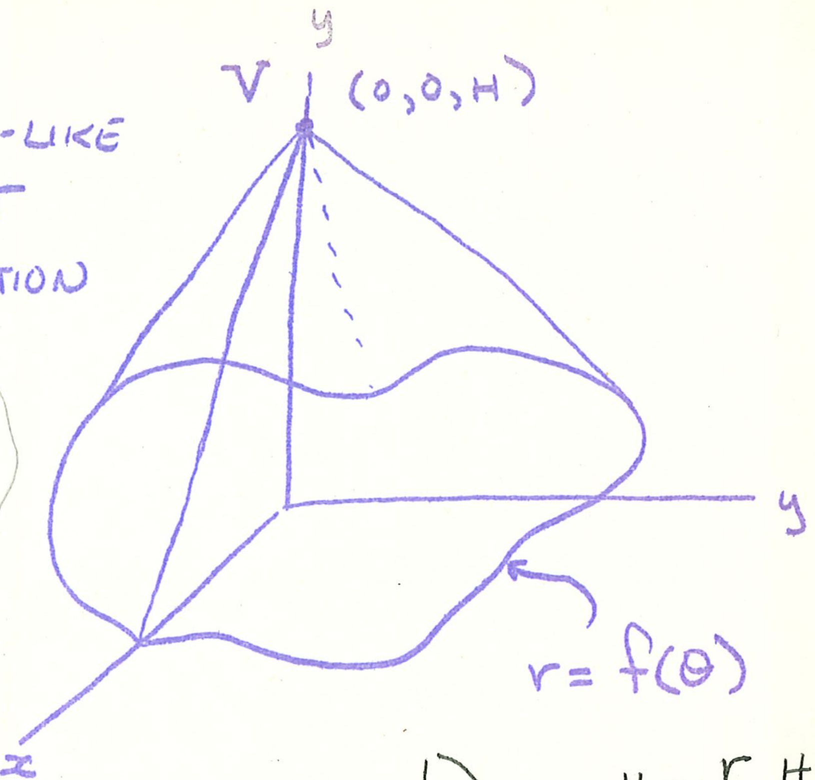
{HINT: SHOW $A_{xy} = \mathcal{A}\cos\gamma$, $A_{yz} = \mathcal{A}\cos\alpha$, etc.}

B) CHECK YOUR ANSWER IN PROBLEM 1 BY USING

10 THE RESULT OF PART A ABOVE.

VI

SHOW THAT THE VOLUME OF THE CONE-LIKE OBJECT TO THE RIGHT SATISFIES THE EQUATION VOLUME = $\frac{1}{3}$ HEIGHT TIMES THE AREA OF THE BASE. (HERE H = HEIGHT.)



~~10~~

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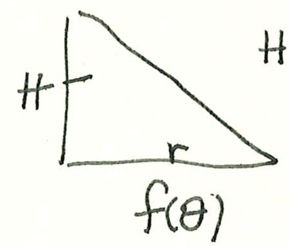

$H - \frac{r}{f(\theta)} H$

$r = \int$

$$B = \int_0^{2\pi}\int_0^{f(\theta)} r \, dr \, d\theta$$

$$= \int_0^{2\pi} \frac{f^2(\theta)}{2} \, d\theta$$

$$V = \int_0^{2\pi}\int_0^{f(\theta)}\int_0^{H\left(1-\frac{r}{f(\theta)}\right)} r \, dz \, dr \, d\theta$$

$$= \int_0^{2\pi}\int_0^{f(\theta)} H\left(r - \frac{r^2}{f(\theta)}\right) dr \, d\theta$$

$$= H\int_0^{2\pi} \left.\frac{r^2}{2} - \frac{r^3}{3f(\theta)}\right|_0^{f(\theta)} d\theta = H \int_0^{2\pi} \frac{f^2(\theta)}{6} \, d\theta$$

MATH 32 FINAL

USE <u>ONE</u> SIDE OF PAPER; SHOW <u>ALL</u> WORK; NEATNESS AND READABILITY WILL COUNT

1. FIND THE POINT OF INTERSECTION OF THE LINE: $\vec{x} = \langle 2,0,3 \rangle + t\langle 3,4,5 \rangle$ AND THE PLANE: $7x - 3y - 2z = 47$.

2. FIND THE MASS OF THE SOLID BOUNDED BY THE SURFACES $z = xy$, $x = 1$, $y = x$, $z = 0$ WITH A DENSITY $\delta = 1 + 2z$.

3. FOR WHAT x DO THE FOLLOWING SERIES CONVERGE?

A. $\displaystyle\sum_{n=0}^{\infty} x^n$

B. $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{3^n n^3}$

C. $\displaystyle\sum_{k=0}^{\infty} \frac{x^k}{k!}$

D. $\displaystyle\sum_{j=0}^{\infty} j!\, x^j$

4. FOR THE SCALAR FIELD $T(x,y,z) = e^{-z^2} \sin x \cos y$ FIND THE GRADIENT AND THE EQUATIONS OF THE TANGENT PLANE AND NORMAL LINE TO THE LEVEL SURFACE AT THE POINT

4. (cont) $(\pi/2, 2\pi, \sqrt{\ln 2})$

5. FOR THE CURVE $\vec{R}(t) = \langle \cos t, \sin t, t \rangle$

FIND: $\vec{V}(t)$, $\vec{A}(t)$, $\vec{T}(t)$, $\vec{N}(t)$, $\vec{B}(t)$, κ, τ, $\frac{ds}{dt}$ and the arclength from $t=0$ to $t=a$.

[FORMULAS: $\vec{V}(t) = \frac{d\vec{R}}{dt}$, $\vec{A}(t) = \frac{d^2\vec{R}}{dt^2}$, $\frac{ds}{dt} = |\vec{V}(t)|$

$\vec{T}(t) = \frac{\vec{V}(t)}{|\vec{V}(t)|}$, $\kappa \vec{N}(t) = \frac{d\vec{T}}{ds} = \frac{d\vec{T}}{dt}\frac{dt}{ds}$, $\vec{B}(t) = \vec{T}(t) \times \vec{N}(t)$

$-\tau \vec{N} = \frac{d\vec{B}}{ds}$, $|\vec{N}(t)| = 1$] [arclength $= \int_0^a \frac{ds}{dt}\,dt$]

6. FIND THE SURFACE AREA OVER THE REGION $a \le r \le b$, $0 \le \theta \le \pi/2$ OF THE CONE $z = mr$.

7. SHOW THAT THE TAYLOR SERIES FOR e^x CONVERGES UNIFORMLY TO e^x ON THE INTERVAL $[-L, L]$ = set of x with $-L \le x \le L$.

8. A SHOW THAT $z = f(x^2 - y^2)$ SATISFIES THE EQUATION $x\frac{\partial z}{\partial y} + y\frac{\partial z}{\partial x} = 0$

B. FIND $\partial w/\partial t$ GIVEN $w = e^{x+2y}\sin(2x - y)$

$x = t^2 + 2u^2$ $\quad y = 2t^2 - u^2$

9. A. DEFINE $\vec{A} \times \vec{B}$

B. SHOW WHY $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

C. FIND VECTORS $\vec{A}$, $\vec{B}$, and $\vec{C}$ SUCH THAT $(\vec{A} \times \vec{B}) \times \vec{C} \neq \vec{A} \times (\vec{B} \times \vec{C})$

D. FIND THE EQUATION OF THE PLANE THAT PASSES THROUGH THE ORIGIN AND CONTAINS THE VECTORS $\vec{A}$ AND $\vec{B}$.

10. PROVE <u>ONE</u> OF THE FOLLOWING:

I: IF $u(x)$ IS THE UNIFORM LIMIT OF THE SERIES $\sum_{k=0}^{\infty} u_k(x)$ ON THE INTERVAL $[a,b]$ AND EACH $u_k(x)$ IS CONTINUOUS THEN $u(x)$ IS CONTINUOUS

II: IF $\vec{R(t)}$ IS A CURVE: $\frac{d\vec{R}}{dt} \cdot \vec{R} \equiv 0$ IF AND ONLY IF $|\vec{R}|$ IS A CONSTANT.

III: IF $u(x)$ IS THE UNIFORM LIMIT OF THE SERIES $\sum_{k=0}^{\infty} u_k(x)$ ON THE INTERVAL $[a,b]$ THEN $\int_a^b u(x)\,dx = \sum_{k=0}^{\infty} \int_a^b u_k(x)\,dx$

IV FOR ANY TWO VECTORS $\vec{x}$ AND $\vec{y}$

$$|x-y|^2 + |x+y|^2 = 2(|x|^2 + |y|^2)$$