MAC 3313 — Calculus 3

Section 3, Spring 1996. MWF 12:20–1:10 TR 12:30–1:45 102 LOVE

Instructor: Bellenot.

The good doctor's Office: 002-B Love, Office Hours: MWF 2:30 – 3:15 or by appointment. Email addressed bellenot@math.fsu.edu will get to the good doctor, who reads his email often.

Eligibility: A grade of C- or better in Calculus 2 (MAC 3312).

Text: Steward 2^{nd} edition. (There are two ways to get the text. If you don't have a copy, the *Multivariable Calculus* version covers what you need for this class. There is no need to buy the version that includes all of calculus.)

Coverage: Chapters 11-14 and additional material on Maple/Mathematica as time allows.

Final: At 10:00 – 12:00 Tuesday, Apr 23, 1996. (Great final time.)

Tests: (3) Tentatively at Jan 30(31?), Feb 27(28?) (or Mar 5(6?)) and Apr 9(10?). No Makeup tests.

Quizes and/or Projects: Every Wednesday. No Makeup quizes. No Late Projects.

Grades: 90% A, 80%B, 70%C, 60%D.

Relative Weights F = 2T and T = Q&P (F is 1/3, each T is 1/6 and Q&P is 1/6).

Homework and Attendance are required. Indeed attendance will be taken by checking off homework. It is the student's responsibility to see that homework is delivered on time. (The homework needs to be turned in even when the student is absent.) Likewise, being absence is not a valid reason for not knowing the next assignment.

Five or more late or missing homeworks is an automatic FAIL.

Fair Warning: *The good doctor teaches this course differently than other instructors. The order of material will be different and so will be the emphasis.*

Projects: The computer packages Mathematica and Maple will be used for projects in this section only.

The Web page for the class is "http://www.math.fsu.edu/~bellenot/class/cal3".

Mathematica
1. Start Mathematica with the 'xmath' command. (SHIFT+RETURN)
2. Define a function.
```
f[x_,y_]:=x^2 + y^2
```
3. Plot a function
```
Plot3D[f[x,y],{x,-1,1},{y,-1,1}]
```
4. Personalize the plot
```
Plot3D[f[x,y],{x,-1,1},{y,-1,1},PlotLabel->"The Good Doctor"]
```
5. Select the plot (with the mouse)
6. File menu, print selection.
7. Print dialog, print to file (say to the file "foo.ps")
8. Print the file (outside of Mathematica, at the taylor prompt) lpr foo.ps

Other options to try. (like the PlotLabel in 4)
A. PlotPoints->{10,10} and PlotPoints->{50,50}
B. Boxed->False
C. // maple Used the Style and Color menu to change things. (Click with middle mouse
   button for the re-draw.)
D. ViewPoint->{-2.4, 1.3, 2.0 } // changes view
E. Use help -> open function Browser
   select Graphics and Sound ->3DPlots->Options

Maple
1. Start Maple with the 'xmaple' command. (NEED ';' to end commands and RETURNS)
2. Define a function
```
f:=(x,y)->x^2 + y^2;
```
3. Plot a function
```
plot3d(f(x,y),x=-1..1,y=-1..1);
```
4. Personalize the plot (Note the 'backquotes')
```
plot3d(f(x,y),x=-1..1,y=-1..1,title=`The Good Doctor`);
```
5. On the pop-up graphics. File menu, print submenu select Postscript.
6. Print dialog prints to a file. (say "foo.ps")
7. Print the file (outside of Maple, at the taylor prompt) lpr foo.ps

Other options to try. (like the title in 4)
A. grid=[10,10] and grid=[50,50]
B. axes=boxed
C. Used the Style and Color menu to change things. (Click with middle mouse
   button for the re-draw.)
D. Drag with left mouse button for view change.
E. Use help -> help browser
   select graphics->3D->Options->other options to find other things to do
   with plot3d (see also graphics->3D->plot3d)

For project 1, I want one "NICE" plot of the function
```
g(x,y)= 3*(1-x)^2*exp(-x^2-(y+1)^2)
        - 10*(x/5-x^3-y^5)*exp(-x^2-y^2) -(1/3)*exp(-(x+1)^2-y^2)
```
(NOTE it takes both lines to define g(x,y))
personalized with your name, your plot points, your viewpoint.
from both Mathematica and Maple.

This project uses your choice of either Mathematica or Maple (And not both.) The project requires you to print 5 graphs.

hmmm what should they be?
perhaps a pair of intersecting implicitplots, like a hyperboid and a plane.
perhaps a space curve and the surface it lives on
perhaps a level curve of the function from project 1
perhaps a curve in cylinder or spherical co-ordinates
finally something that you have to work at to get the correct picture.

1. Graph x^2+xy+2y^2+7yz-z^2+3x-xz=1 with axis labeled correctly and identify the figure.
2. The curve <cos(e^t),sin(e^t),t> t in [0,5] (this graph has a pretty picture and the initial graph is not.)
3. ContourPlot of the function from project 1
4. Cylinder co-ordinates r=1+z sin(theta) for z in [0,2]
5. Looking Problem 35 Section 11.6, it follows that the intersection of the hyperbolic parabolid z=x^2-y^2 and the plane y=x+1 is a straight line. Graph the two curves together so we can see this line.

For reference here is the old handout given in the lab
This is the function for project one. [Note it is not how it is entered in Maple nor Mathematica.]

```
g(x,y)= 3*(1-x)^2*exp(-x^2-(y+1)^2)
        - 10*(x/5-x^3-y^5)*exp(-x^2-y^2) -(1/3)*exp(-(x+1)^2-y^2)
```

Maple

```
with(plots);
implicitplot3d(x^2+y^2+z^2=1,x=-2..2,y=-2..2,z=-2..2);

x^2-y^2+z^2=1
x^2-y^2-z^2=1
x-y^2-z^2=1
x-y^2-z^2=0
x-y^2+z^2=0

with(plots);
contourplot(x^2-y^2,x=-2..2,y=-2..2);

with(plots);
spacecurve([sin(t),cos(t),t],t=0..4*Pi);
```

Mathematica

```
Needs["Graphics`ContourPlot3D`"] [Note double quotes and backquotes]
or <<"Graphics/ContourPlot3D.m"
ContourPlot3D[x^2+y^2+z^2-1,{x,-2,2},{y,-2,2},{z,-2,2}]

ContourPlot[x^2-y^2,{x,-2,2},{y,-2,2}]


ParametricPlot3D[{Sin[t],Cos[t],t},{t,0,4*Pi}]


Needs["Graphics`ParametricPlot3D`"] [Note double quotes and backquotes]
or <<"Graphics/ParametricPlot3D.m"
```

This project has 4 parts. All plots need your name as part of the title.

Part 1. A plot of the surface x^2+y^2+z^2=49 and its tangent plane at (6,2,3) include (labeled) axes. The tangent plane needs to show as a plane (that is, do not pick a view where it is a line.) [The normal to the tangent plane is given by grad F.]

Part 2. A 3D gradient plot of x^2+y^2-z^2 with (labeled) axes and with the graph oriented in a readable manner. [maple has gradplot and gradplot3d, mathematica has PlotGradientField and PlotGradientField3D.]

Part 3. A 2D plot which shows both the gradient and the level surfaces of f(x,y) = x^2 - y^2 on the same graph (see help below on how to do this.)

Part 4. Repeat part 3 on the function from project 1.

```
g(x,y)= 3*(1-x)^2*exp(-x^2-(y+1)^2)
        - 10*(x/5-x^3-y^5)*exp(-x^2-y^2) -(1/3)*exp(-(x+1)^2-y^2)
```

maple: To combine two 2D plots try:

```
a:=gradplot(x^2+y^2,x=-5..5,y=-5..5):
                                    ^ note the : and not ; try it both ways.
b:=implicitplot(x^2+y^2=16,x=-5..5,y=-5..5):
display({a,b}); <- back to semi-colon.
```

mathematica: To combine two 2D plots try:

```
<<"Graphics/PlotField.m"
a=PlotGradientField[x^2+y^2,{x,-5,5},{y,-5,5}];
b=Plot[x,{x,-5,5}];
Show[a,b]
```

Final form, nothing changed, I just added an example.

This project has 4 parts. All plots need your name as part of the title. Each part has 2 plots, a 3d plot with the axis labeled and a combined density and contour plot. All plots are over the range 0 - 2pi by 0 - 2pi.

Part 1. sin(x+y)

Part 2. sin(xy)

Part 3. sin(x)+sin(y)

Part 4. sin(x)sin(y)

The following example will help maple users

```
f:=(x,y)->x^2+y^2;
L:={seq(f(x,y)=2*i,i=1..9)};
a:=implicitplot(L,x=-3..3,y=-3..3):
b:=densityplot(x^2+y^2,x=-3..3,y=-3..3):
display({a,b});
```

For once, mathematica users win, this is the default behavior for ContourPlot.

```
ContourPlot[x^2+y^2,{x,-3,3},{y,-3,3}]
```

From proj3.txt
maple: To combine two 2D plots try:

```
a:=gradplot(x^2+y^2,x=-5..5,y=-5..5):
                                    ^ note the : and not ; try it both ways.
b:=implicitplot(x^2+y^2=16,x=-5..5,y=-5..5):
display({a,b}); <- back to semi-colon.
```

mathematica: To combine two 2D plots try:

```
<<"Graphics/PlotField.m"
a=PlotGradientField[x^2+y^2,{x,-5,5},{y,-5,5}];
b=Plot[x,{x,-5,5}];
Show[a,b]
```

This project has 5 parts. All plots need your name as part of the title. Each plot is of one or more 2d vector fields F = <P(x,y),Q(x,y)>, and the equation of the Vector Field needs to be in the title also. If not otherwise given use the range -2..2 for both x and y.

Part 1. This part has several plots, but only one of the plots is to be printed. Here P = k * y and Q = k * x for the three values of k = 1, 2, 8. (Print the one that looks the best.)

Part 2. Minor changes in P or Q can make major changes in the "global look" of the Vector Fields plot. Take P = -y, Q = x which just negates the P term from part 1.

Part 3. Take the vector field from part 2 and normalize the F an each point and plot it. (that is replace F by F/|F| where |F| = length of F.)

Part 4. Actually most of any vector field looks very boring. Plot the vector field of part 2 over some ranges like x=k..k+2,y=m..m+2 for (k,m)=(100,100), (200,100) and (100,200). Only print one of the three plots, the one where the arrows are "most pointed up".

Part 5. The interesting parts of a vector field are near points where the vector field is zero. (We have seen this before, y=f(x) does its interesting things where f'(x) is zero.) There are only a certain number of behaviors near a zero, we have seen two above. Look at F=<x,y>, F=<-x,-y>, F=<-x,y>, F=<-x,0> and print the one that looks most like an explosion.

The following example will help maple users

```
with(plots);
p:=y:q:=-sin(x)-y/10:
fieldplot([p,q],x=-10..10,y=-3..3);
```

mathematica users

```
<<"Graphics/PlotField.m"
f[x_,y_]:={y,-Sin[x]-y/10}
PlotVectorField[f[x,y],{x,-10,10},{y,-3,3}]
```

This project is now ready

This project has 4 parts. All plots need your name as part of the title. Each plot is of one or more surfaces given by parametric equations r(u,v) = <x(u,v),y(u,v),z(u,v)>, and the equation of the surface needs to be in the title also. If not otherwise given use the range -2..2 for both u and v.

Part 1. Part of the sphere of radius 4 over u,v=0..1

Part 2. <u cos v, u sin v, u> u=0..1, v=0..2pi

Part 3. < cosh u * cos v, cosh u * sin v, u> u =-2..2, v=0..2pi.

Part 4. <cos u (cos 3u/2 + 2 + 0.3 cos v), sin u (cos 3u/2 + 2 + 0.3 cos v), sin 3u/2 + 0.3 sin v > u=0..4pi, v=0..2pi. (Use "Torus Knot" for title)

The following example will help maple users
This project uses plot3d so you don't have to say with(plots);

```
A:=(u,v)->(3+cos(v))*cos(u)
B:=(u,v)->(3+cos(v))*sin(u)
C:=(u,v)->sin(v)
plot3d([A(u,v),B(u,v),C(u,v)],u=0..2*Pi,v=0..2*Pi);
```

mathematica users
This project uses ParametricPlot3D, load it with

```
"<<Graphics/ParametericPlot3D.m"
A[u_,v_]:=(3+Cos[v])*Cos[u]
B[u_,v_]:=(3+Cos[v])*Sin[u]
C[u_,v_]:=Sin[v]
ParametricPlot3D[{A(u,v),B(u,v),C(u,v)},{u,0,2*Pi},{v,0,2*Pi}]
```

MAC 3313 Calculus 3 **Test 1** 31 Jan 1996

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper.

1. If the position function is $\mathbf{r}(t) = \left\langle t^3, t^2+1, t^3-1 \right\rangle$, find the velocity, the speed and the acceleration.

2. Find the equation of the plane through the point $(6, 5, -2)$ parallel to the plane $x + 2y - z + 1 = 0$.

3. Find the point where the line $x = 1 + t, y = 2t, z = 3t$ intersects the plane $3x - 2y + z = 9$.

4. Find the value of x such that the vectors $\langle 2, x, 3\rangle$ and $\langle x, 8, 6\rangle$ are perpendicular and find the value x such the vectors are parallel.

5. Find the scalar and vector projections of $\mathbf{b} = \mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$ onto $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} - 6\mathbf{k}$.

6. Find parametric equations for the line of intersection of the planes $2x + 5z = -3$, and $x - 3y + z = -2$.

7. Identify and sketch the graph of the equation $x^2 + y^2 + z^2 = 2x$ and re-write the equation in both cylinderical and spherical co-ordinates.

6. Find the equation of the plane that passes through the point $(0, 1, 2)$ and contains the line $x = y - 1 = z$.

9. Find and simplify both the unit tangent vector $\mathbf{T}(t)$ and the curvature $\kappa = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$ of the space curve $\mathbf{r}(t) = \left\langle t^2, 2t^3/3, t \right\rangle$.

10. Find and simplify the arclength of $\mathbf{r}(t) = \langle e^t, e^t sin(t), e^t cos(t)\rangle$, $0 \le t \le 2\pi$.

MAC 3313 Calculus 3 **Test 2** 29 Feb 1996

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper.

1. Find the equation of the tangent plane to $f(x,y) = x^2 + y^2$ at $(3,4)$.

2. Use the chain rule to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ if $z = xe^y + ye^{-x}$, $x = e^t$ and $y = st^2$

3. Set up but do **NOT** evaluate the interated integral (or sum of interated integrals) for the volume under the surface of $z = xy\cos(x+y) + e^x\sqrt{2y+8}$ and above the region bounded by $x = y^2$ and $x + y = 2$.

4. Find the directional derivative of $f(x,y,z) = \sqrt{xyz}$ at the point $(2,4,2)$ in the direction of the vector $\langle 4, 2, -4 \rangle$.

5. The function $f(x,y) = x^3 - 3xy + y^3$ has a pair of critical points find them and determine if they are local minimums, local maximums or saddle points.

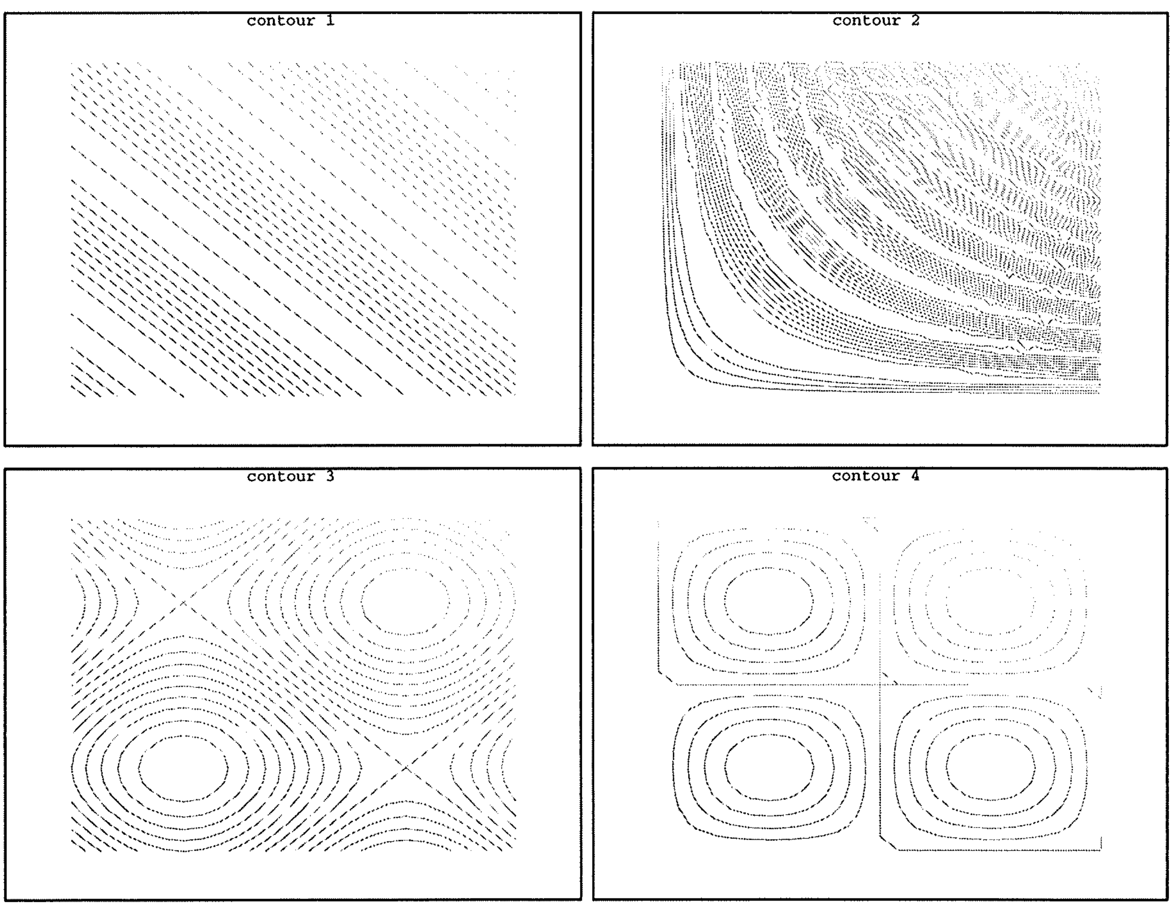
6. Show the limit $\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+2y^2}$ does not exist.

7. Sketch the region of integration and change the order of integration of $\int_1^2 \int_0^{\ln x} f(x,y)\, dydx$

8. Use Lagrange multiplers to find the maximum and minimum values of $f(x,y) = x^2 - y^2$ subject to the constraint $x^2 + y^2 = 4$.

9. Compute the mass of the lamina of the region in the first quadrant inside $x^2 + y^2 = 9$ and outside $x^2 + y^2 = 1$ with density $\rho(x,y) = e^{-(x^2+y^2)}$. Polar co-ordinates might come in handy.

10. Below are maple contour plots of the functions (in some order) of $\sin(x)\sin(y), \sin(xy), \sin(x) + \sin(y)$ and $\sin(x+y)$ Identify which is which. The plots are over $[0, 2\pi] \times [0, 2\pi]$.



Maple contour plots

MAC 3313 Calculus 3 **Test 3** 9 Apr 1996

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper.

1. Find the curl and div of $\mathbf{F} = \langle x^2y, yz^2, zx^2 \rangle$.

2. Find f so that $\mathbf{F} = \nabla f$ and use it to find the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$. Here $\mathbf{F} = \langle 2xz + \sin y, x \cos y, x^2 \rangle$ and C is a curve from $(1,0,0)$ to $(1,0,2\pi)$.

3. Evaluate the line integral $\int_C x^2y dx - 3y^2 dy$ using Green's Theorem when C is the curve which goes around the perimeter of the region $\{(x,y) : 0 \le x \le 1, 0 \le y \le 1\}$ in the backwards (clockwise) direction.

4. Find the equation of the tangent plane to the parametric surface given by $\langle u^2, u - v^2, v^2 \rangle$ at the point $(1,0,1)$.

5. Rewrite but do **NOT** evaluate the surface integral $\int\int_S \mathbf{F} \cdot d\mathbf{S}$ as an usual double iterated integral (including limits of integration and a simplified integrand). Here $\mathbf{F} = \langle y, x, xy \rangle$ and S is the portion of the paraboloid $z = x^2 + 2y^2$ over the region $\{(x,y) : 1 \le x \le 2, \ln x \le y \le \pi\}$ Use the upward pointing normal of S.

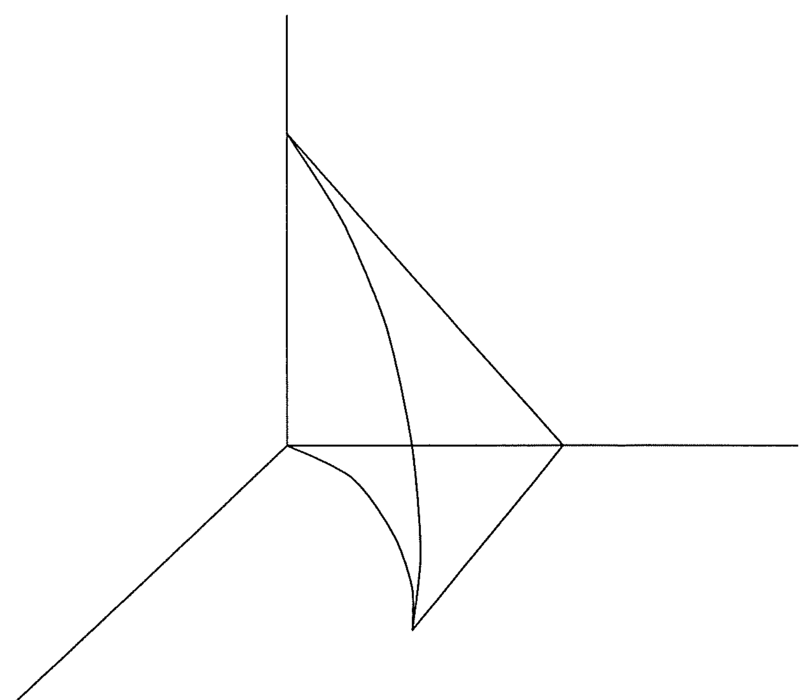
6. Set up but do **NOT** evaluate a double iterated integral for the surface area of the surface $z = y^2 - x^2$ that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$. The double iterated integral needs to have limits of integration and a simplified integrand.

7. Use cylindrical co-ordinates to evaluate $\int\int\int_E x^2 dV$ when E is the solid within $x^2 + y^2 = 1$, above $z = 0$ and below $z^2 = 4x^2 + 4y^2$.

8. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ if $\mathbf{F} = \langle x^2y, -xy \rangle$, and $\mathbf{r}(t) = \langle t^3, t^4 \rangle, 0 \le t \le 1$.

9. Use the given transformation to evaluate $\int\int_R x dA$ where R is the region in the **FIRST** quadrant where $9x^2 + 4y^2 \le 36$ and the transformation is $x = 2u, y = 3v$. Also explicitly draw R and S, the region in the uv plane that maps to R in the xy plane by this transformation. Clearly label the Jacobian of the transformation.

10. Rewrite the the limits of $\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x,y,z) dz dy dx$ in the orders $dxdydz$ and $dydzdx$.



Hint

MAC 3313 Calculus 3 Section 3

Class Handout

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1. M 8 Jan Homework 11.1: 1 - 45 = 1 mod 4.
2. T 9 Jan Homework 11.2: 1 - 29 = 1 mod 4.
3. W 10 Jan Homework 11.3: 1 - 49 = 1 mod 4. Quiz 1
4. R 11 Jan No Class
5. F 12 Jan Computer Accounts, Intro to Mathematica/Maple

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6. M 15 Jan No Class
7. T 16 Jan Homework 11.4 1 - 33 = 1 mod 4; 11.5 1 - 13 = 1 mod 4
8. W 17 Jan Project 1 assigned -- Visit to the ulab.
9. R 18 Jan Homework 11.5 17 - 57 = 1 mod 4; 11.6 1 - 17 = 1 mod 4 Quiz 2
10. F 19 Jan Homework 11.6 21 - 37 = 1 mod 4;

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11. M 22 Jan Homework 11.7 1 - 45 = 1 mod 4
12. T 23 Jan Class At Ulab.
13. W 24 Jan Homework 11.8 1 - 29 = 1 mod 4. Project 1 Due.
14. R 25 Jan Homework 11.9 1 - 17 = 1 mod 4.
15. F 26 Jan Homework 11.9 27,29; 11.10 1 - 39 odd

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16. M 29 Jan Homework 11.10 51-63 odd
17. T 30 Jan Review - Project 2 handout
18. W 31 Jan Test 1
19. R 1 Feb No Class
20. F 2 Feb Homework 12.1 1 - 55 = 1 mod 4; 59 - 64

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21. M 5 Feb Homework 12.2 1 - 41 = 1 mod 4
22. T 6 Feb Homework 12.3 1 - 77 = 1 mod 4
23. W 7 Feb Homework 12.4 1 - 33 = 1 mod 4 Project 2 Due
24. R 8 Feb No Class
25. F 9 Feb Homework 12.5 1 - 45 = 1 mod 4

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26. M 12 Feb Homework 12.6 1 - 45 = 1 mod 4
27. T 13 Feb Homework 12.7 1 - 17 = 1 mod 4 Project 3 handout.
28. W 14 Feb Homework 12.7 21 - 37 = 1 mod 4 Quiz 3
29. R 15 Feb Homework 12.8 1 - 9 = 1 mod 4
30. F 16 Feb Homework 12.8 13

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31. M 19 Feb Homework 13.1 3; 13.2 1 - 7 odd.
32. T 20 Feb No Class
33. W 21 Feb Homework 13.2 13 - 21 = 1 mod 4; 13.3 1 - 17 = 1 mod 4 Project 3 Due
34. R 22 Feb Homework 13.3 21 - 45 = 1 mod 4; 13.4 1 - 13 = 1 mod 4
35. F 23 Feb Homework 13.4 17 - 29 = 1 mod 4

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36. M 26 Feb Homework 13.5 1 - 13 = 1 mod 4
37. T 27 Feb Homework 13.5 17, 21
38. W 28 Feb Review
39. R 29 Feb Test 2
40. F 1 Mar No Class

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41. M 4 Mar Homework 13.6 1 - 13 = 1 mod 4
42. T 5 Mar Homework 13.7 1 - 25 = 1 mod 4
43. W 6 Mar Homework 13.7 29 - 37 = 1 mod 4
44. R 7 Mar No Class
45. F 8 Mar Homework 13.8 1 - 33 = 1 mod 4Quiz 4

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46. M 11 Mar Homework 13.9 1 - 13 = 1 mod 4
47. T 12 Mar Homework 13.9 17, 21; 14.1 1 - 17 = 1 mod 4.
48. W 13 Mar Homework 14.2 1
49. R 14 Mar Homework 14.2 5 - 29 = 1 mod 4;Project 4 Due
50. F 15 Mar No Class

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51. Spring Break

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52. M 25 Mar No Class
53. T 26 Mar Homework 14.3 1, 5, 9, 25
54. W 27 Mar Homework 14.3 13, 17, 21, 29. Quiz 5
55. R 28 Mar Homework 14.4 1 - 17 = 1 mod 4.
56. F 29 Mar Homework 14.4 21, 25; 14.5 1 - 21 = 1 mod 4.

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57. M 1 Apr Homework 14.5 20, 22, 31 - 38.
58. T 2 Apr Homework 14.6 1 - 21 = 1 mod 4
59. W 3 Apr Homework 14.7 1 - 25 = 1 mod 4
60. R 4 Apr No Class
61. F 5 Apr Homework 14.6 25, 26; 14.7 29, 33. Project 5 Due

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62. M 8 Apr Review
63. T 9 Apr Test 3
64. W 10 Apr No Class
65. R 11 Apr No Homework
66. F 12 Apr Homework 14.8 1 - 17 = 1 mod 4

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67. M 15 Apr Homework 14.9 1 - 13 = 1 mod 4
68. T 16 Apr Project 6 Due
69. W 17 Apr Review
70. R 18 Apr No Class
71. F 19 Apr Review

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72. T 23 Apr -- Final 10:00 - 12:00

last update 15 Apr 96.