Math 3p1 Test III April 30, 1973

1) (30 pts) Find the following limits

a) $\lim_{x \to 0} (\cos x)^{1/x}$

b) $\lim_{x \to 0^+} x^{1/\ln x}$

2) (20 pts) Consider the curve given by $x(t) = \ln t$, $y(t) = t$. Find the length of the curve mapped out as t moves over the interval $[1, \infty)$.

3) (20 pts) Find the maximum and minimum values of curvature for the curve given by $y = 1/x$.

4) (15 pts) Evaluate $\int_1^\infty \frac{1}{x^2} \sin \frac{1}{x} \, dx$.

5) (15 pts) Evaluate $\int_1^\infty \left(\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{x+1}} \right) dx$.

MATH 31 TEST IV

1 FOR THE CURVE $\vec{R}(t) = \langle e^t \cos t, e^t \sin t \rangle$

FIND:

a) THE VELOCITY $\vec{V}(t)$,

b) THE SPEED $\frac{ds}{dt}$,

c) THE TANGENT VECTOR $\vec{T}(t)$,

d) THE ARCLENGTH FROM $t=0$ to $t=2\pi$.

2 FIND THE FOLLOWING LIMITS AND INTEGRALS

a) $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$

b) $\lim_{x \to -\infty} x^2 e^x$

c) $\int_1^\infty \frac{dx}{1+x^2}$

d) $\int_0^\infty x e^{-x} dx$

e) $\lim_{x \to 3} \frac{x^2 - 4x + 3}{x^2 + x - 12}$

f) $\lim_{x \to 1} \frac{x^3 - 3x + 1}{x^4 - x^2 - 2x}$

MATH 31 TEST V

USE ONE SIDE OF PAPER; NEATNESS AND READABILITY WILL COUNT; SHOW ALL WORK.

TEST THE CONVERGENCE OF THE SERIES IN 1-4

1. $\sum_{n=0}^{\infty} (-1)^n$

2. $\sum_{k=1}^{\infty} \frac{1}{2^k+1}$

3. $\sum_{j=3}^{\infty} \frac{1}{\ln j}$

4. $\sum_{m=0}^{\infty} \frac{m^2}{m!}$

5. FIND THE SUM OF THE INFINITE SERIES

$$\pi + \frac{\pi}{\sqrt{2}} + \frac{\pi}{\sqrt{2^2}} + \frac{\pi}{\sqrt{2^3}} + \cdots + \frac{\pi}{\sqrt{2^n}} + \cdots$$

6. FIND THE AREA INSIDE $r = +\sqrt{1+\sin\theta}$

7. CHANGE $r = 4\sec\theta$ INTO RECTANGULAR CO-ORDINATES.

8. FOR WHAT VALUES OF x DOES THE POWER SERIES $\sum_{n=0}^{\infty} \frac{n x^n}{2^n}$ CONVERGE?

9. STATE THE CAUCHY INTEGRAL TEST AND USE IT ON $\sum_{n=3}^{\infty} \frac{1}{n \ln n}$

10. FIND THE TAYLOR SERIES ABOUT $x=0$ FOR $\tan^{-1} x$ (ALSO CALLED THE MACLAURIN SERIES.)

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MATH 31 FINAL

SHOW ALL WORK; NEATNESS AND READABILITY WILL COUNT.

1. FIND EQUIVALENT RATIONAL FRACTION FOR THE FOLLOWING INFINITE REPEATING DECIMAL FRACTIONS:

A. ~~0.3333...~~ $0.77777\ldots$

B. ~~$0.681111\ldots$~~ $0.47111\ldots$

C. ~~$5.878787\ldots$~~ $5.474747\ldots$

2. FIND THE FOLLOWING INTEGRALS

A. ~~$\int e^{\tan x} \sec^2 x\,dx$~~ $\int e^x \sin e^x\,dx$

B. ~~$\int e^x \cos x\,dx$~~ $\int \ln x\,dx$

C. ~~$\int \frac{2\,dx}{x^2-1}$~~ $\int \frac{4\,dx}{x^2-3x+2}$

3. TEST THE CONVERGENCE OF THE FOLLOWING (STATE WHY)

A. ~~$\sum_{k=0}^{\infty} \frac{1}{k+5}$~~ $\sum_{k=0}^{\infty} \frac{1}{\sqrt{n^3-5}}$

B. $\sum_{n=2}^{\infty} \frac{1}{\ln n}$

C. ~~$\sum_{n=1}^{\infty} \frac{n!}{n^n}$~~ $\sum_{n=1}^{\infty} \frac{n^n}{n!}$

4. DERIVE THE FORMULA

$$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}.$$

5. FOR THE CURVE $\vec{R(t)} = \langle a\cos t, b\sin t \rangle$:

A; SHOW $\vec{R(t)}$ LIES ON THE ELLIPSE

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

B. FIND ~~A FORMULA FOR THE CURVATURE OF $\vec{R(t)}$ AS A FUNCTION OF t.~~ VELOCITY AND TANGENT VECTOR

C. SET UP - BUT DO NOT TRY TO EVALUATE - THE INTEGRAL FOR THE ARCLENGTH FROM $t = $ ~~0~~ 1 TO $t = d$.

6. FIND THE FOLLOWING IMPROPER INTEGRALS

A. $\int_0^\infty xe^{-x}\,dx$ B. $\int_{-1}^{1} \frac{dx}{x}$

7. FIND THE AREA OF THE REGION ~~INSIDE~~ OUTSIDE THE CARDIOID $r = 2a(1+\cos\theta)$ AND ~~OUTSIDE~~ INSIDE THE CIRCLE $r = $ ~~$3a$~~ $6a\cos\theta$ [HINT: SKETCH THE GRAPHS.]

8. FIND THE TAYLOR SERIES ABOUT $x = 0$ FOR $\ln(1-x)$. USE THIS SERIES TO FIND THE INTEGRAL $\int_0^1 \ln(1-x)\,dx$ AS AN INFINITE SERIES OF CONSTANTS.

9. FIND THE FOLLOWING LIMITS

A. $\lim_{x \to 0} \frac{\sin x}{x}$

B. $\lim_{x \to 0^+} x \ln x$

C. $\lim_{x \to \infty} \frac{x + \ln x}{x \ln x}$

D. $\lim_{x \to 0^+} x^x$

10. STATE THE RATIO TEST

FIND EXAMPLES OF SERIES THAT

A. CONVERGE BY THE RATIO TEST

B. DIVERGE BY THE RATIO TEST

C. WHERE THE RATIO TEST FAILS.
BUT SERIES CONVERGES

D WHERE THE RATIO TEST FAILS
BUT SERIES DIVERGES