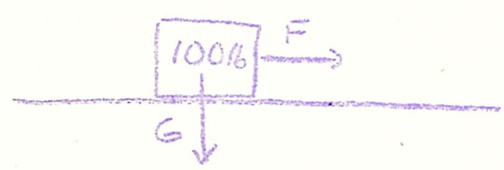
WORK — POWER — & YOUR ELECTRIC BILL
guide, supplement, and motivation for § 7.9
(or who says math is not relevant)

WORK: (this is a definition from physics, and need not bare any relation to the eight-to-five-grind (see examples below)) The amount of work done in a physical act is the force (in direction of motion) times the distance moved.

EXAMPLES 1. If you lift a 10 lb block 10 ft the amount of work = 100 lb-ft, or 100 lb block 1 ft the work is 100 ft-lb. However, If you hold a 1000 lb block over your head for 18 years you have done no work (no motion.)

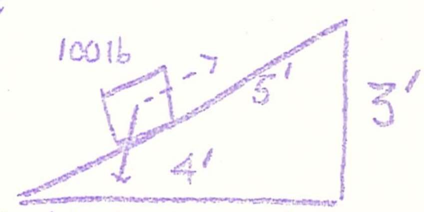
2.



If you pull this block 10 ft in the direction F The amount of work will be much less than 1000 ft-lbs. This is because it takes less than 100 lbs to move the block, in fact, if μ is the coefficient friction, $F = 100\mu$. (There is no motion in the direction G.)

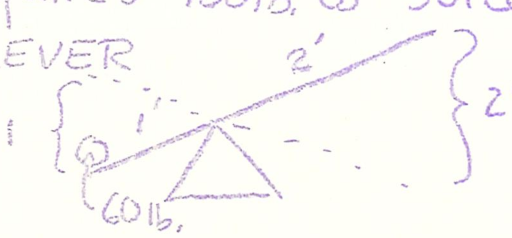
USES 1. SIMPLE MACHINES (or less force, but same work)

A. THE INCLINED PLANE (frictionless of course)



a simple (?) resolution of forces shows that it takes 60 lb in the dotted direction to move the block upward; Hence the work to move the block from the bottom to the top is (60)(5) = 300 ft-lb. This is the same amount of work needed to lift the block 3 ft in the air, but this requires 100lb. of force.

B. LEVER



The Trade-off here is 60 lb of force for 1 ft or 30 lb of force for 2 ft, in either case 60 ft-lb of work is done.

C. PULLEY (COMPOUND (one only changes direction))
this is similar to lever
turn paper sideways →

USES 2. Work-Energy Theorem and the law of conservation of energy. This are important principles in physics (See your friendly physicist) But roughly speaking it is an equivalence between work & energy, and we all know how relevant energy is today.

HOOKE'S LAW & SPRINGS (or those boring problems in the book.) Hooke law says to every spring there is a constant k (called of all things Hooke's constant) such that the force needed to hold the spring x units longer (or shorter) then the rest length is kx.

The amount of work is bit harder to compute since the force varies. But (down to bare bones (see the text for details)) the amount of work done in enlongating a spring from a units longer to b units longer than the rest length is:

$$\int_a^b F(x)\,dx = \int_a^b kx\,dx = \tfrac{1}{2}kx^2\Big|_a^b = \tfrac{1}{2}k(b^2-a^2).$$

So much for the book.

POWER: Work has no sense of time. Which is to say the same amount of work is done lifting a 100 lb block (there are alot blocks arn't there) 3ft is the same if it took 1 min. or 3 years. Thus we have the notion of power which is work per unit time. (equation $P = W/t$). Latter on we will use the equivalent equation $W = P \cdot t$.

UNITS: Power is usually measured in units of horsepower (550 ft-lb/sec (This will made for one tired horse awful fast (though some of the cars I've owned are rated in ponypower (would you believe hamsters?))) or in units of watts (for James Watt (the steam engine)) (Now look at the title again & noted that every electric appliance gives its power requirements in watts) (1 horsepower = 746 watts) or in units of Kilowatts (abv. kw) (1 kw = 1000 watts.) Using the equation in the paragraph before $W = P \cdot t$, A common unit for work in kilowatt-hour (kw.-hr. = using a kw for an hour) (in fact your electric bill is figured on the amount of kw-hr's you use (if you get an electric bill)). We will assume that 1 kw-hr costs 5¢, unless stated otherwise (this is not far off the mark.)

EXAMPLES 1. a 60 watt blub on for 1 hr uses 60 watt-hours or .06 kw-hour and costs about .3¢. Left on for a month = 30 days = 720 hours, it uses 43.2 kw hr at a cost of \$2.16.

VARING POWER: Generally speaking the amout of electric power used is a function of time that is Power = $P(t)$. How much work is done between time $t = a$ to time $t = b$? The answer we will show is $\int_a^b P(t)\,dt$ (you knew integrals would find there way into this sooner or later.) It is enuff to approximate the work by assuming the power is constant (i.e. by $\sum_{i=1}^{n} P(\xi_i)\Delta t_i$) on small intervals and then taking the limit as the intervals get small Q.E.D.

Examples (con't) 2. Suppose the power is given by $P(t) = 4t - t^2$ (watts, no make that kilowatts, for t in hours $0 \le t \le 4$.) How much work is used? It is $\int_0^4 4t - t^2\,dt$

$$= \left(2t^2 - t^3/3\right)\Big|_0^4 = 32 - 64/3 = 32/3 \text{ kw-hr.}$$

3. $P(t) = (t^3-1)^{10}t^2$ $\quad t = 1$ to 10 (kw's & hours)

$$\text{Work} = \int_1^{10} (t^3-1)^{10}t^2\,dt = \frac{1}{3}\int_0^{999} u^{10}\,du = \frac{u^{11}}{33}\Big|_0^{999} = \frac{1}{33}(999)^{11} \text{ kw hr.}$$

PROBLEMS: Each of 1-6, $P(t)$ is given in kilowatts, t in hours, find the amount of work from $t=a$ to $t=b$ and the cost at 5¢/kw hr

1. $P(t) = (24t - t^2)3$ $a=0$ $b=24$
2. $P(t) = t^{1/2} + t$ $a=2$ $b=4$
3. $P(t) = 288$ $a=0$ $b=24$
4. $P(t) = 6(t-12)^2$ $a=0$ $b=24$
5. $P(t) = t(t^2+1)^{-1/2}$ $a=1$ $b=3$
6. $P(t) = -t$ $a=-2$ $b=-1$

7. Suppose the electric company changes its rate system so that it charges 2¢ per kw-hr between $t=0$ & $t=10$ and between $t=19$ & $t=24$; and 10¢ per kw-hr between $t=10$ and $t=19$, Find the new cost in each of 1, 3, 4.

ANS. 1, 3, & 4: $W = 6912$ kw-hr, \$345.60.

2. $W = (34 + 4\sqrt{2})/3$ \$ $(170 + 20\sqrt{2})/300$
5. $W = \sqrt{10} - \sqrt{2}$ \$ $.05(\sqrt{10} - \sqrt{2})$
6. $W = 1\tfrac{1}{2}$ \$.075
7. Cost for *1, \$421.20
*3. \$345.60
*4. \$194.40