MAC 2312 Calculus 2 **Test 2** 8 Mar 2001

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

1. New series from old_1: Write down the Taylor series for $\cos x$ and use it find the Taylor series for $\cos\sqrt{|x|}$. (Make sure your last answer has no square roots or absolute values.) For small positive x, which is bigger $\cos x$ or $\cos\sqrt{x}$?

2. A two meter rod has density function $\rho(x) = x^2 kg/m$ where x is the distance from the left end. Find the mass (in kg) and the center of mass (in m) of the rod.

3. Find both $h'''(0)$. and the radius of convergence of the power series below

$$h(x) = \sum_{n=0}^{\infty} \frac{n^2 x^n}{3^n} = \frac{x}{3} + \frac{4x^2}{9} + \frac{9x^3}{27} + \frac{16x^4}{81} + \dots$$

4. Write the integral and use your calculator's numeric integration to evaluate the integral which yields the arclength of one period of the $\sin(x)$ function.

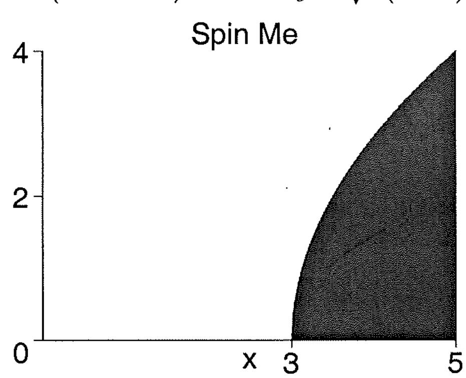
5. New series from old_2: You are given the Taylor series for the functions f and g below.

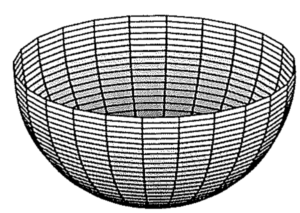
$$f(x) = \sum_{n=1}^{\infty} nx^n = x + 2x^2 + 3x^3 + 4x^4 + \dots$$

$$g(x) = \sum_{n=1}^{\infty} n^2x^n = x + 4x^2 + 9x^3 + 16x^4 + \dots$$

a. Show the long division needed to get the first 4 non-zero terms of f/g.
b. Find the Taylor series for $\int_0^x f(t)/t \quad dt$.

6. Write but do **NOT** evaluate integrals which will compute the volume of the figure obtained by rotating the area (below left) between $y = \sqrt{8(x-3)}$, $3 \le x \le 5$ and the x-axis about A. The x-axis. B. The y-axis.





7. Find the work (in N-m (Newton-meters)) done pumping all the water out of a full fish tank (above right) that has the shape of a hemisphere of radius 1 meter. The density of water, ρ, is $1000kg/m^3$ and g, acceleration due to gravity, is $9.8m/s^2$.

8. For the series $\sum_0^\infty 1/3^n$

A. Find the sum.

B. Show how the ratio test can be used to show the series converges.

C. Show how the integral test can be used to show the series converges.

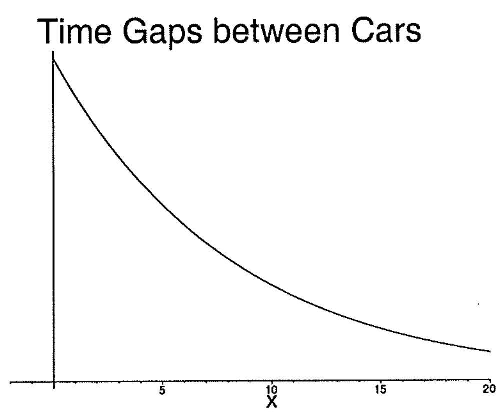
D. Use the series to show $\sum_0^\infty n/(n3^n + \pi)$ converges by the comparison test.

9. Below are three power series that you can assume have radius of converence equal to one. Your job is to check the endpoints x=1 and x=-1 for convergence. Write down the resulting (non-power) series at each endpoint, write down whether each endpoint series converges or diverges and finally give the interval of convergence of the power series. To make the problem more interesting, there are number of other bits of information about each series, but nothing more is required of you.

$$A.\ \sum_1^\infty \frac{(-1)^{n+1}x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots = \int_0^x 1 - t + t^2 + \cdots\ dt = \int_0^x \frac{1}{1+t}\ dt = \ln(1+x)$$

$$B.\ \sum_0^\infty nx^n = x + 2x^2 + 3x^3 + \cdots = x\frac{d}{dx}(1 + x + x^2 + x^3 + \cdots) = x\frac{d}{dx}\frac{1}{1-x} = \frac{x}{(1-x)^2}$$

$$C.\ \sum_1^\infty \frac{x^n}{n^2} = \int_0^x 1 + t/2 + t^2/3 + \cdots\ dt = \int_0^x \frac{t + t^2/2 + t^3/3 + \cdots}{t}\ dt = \int_0^x \frac{\ln(1-t)}{t}\ dt = \text{polylog}(2, x)$$



10. An experiment was done observing the time gaps between cars on Tennessee Street. The data shows that the probability density function of these time gaps was given approximately by $p(x) = \alpha e^{-0.122x}$, where $x \geq 0$ is time in seconds (see above).

a. Find α.

b. Find and sketch P, the cumulative distribution function.

c. Find the median time gap.

d. Find the mean time gap (use TI-89 to integrate).

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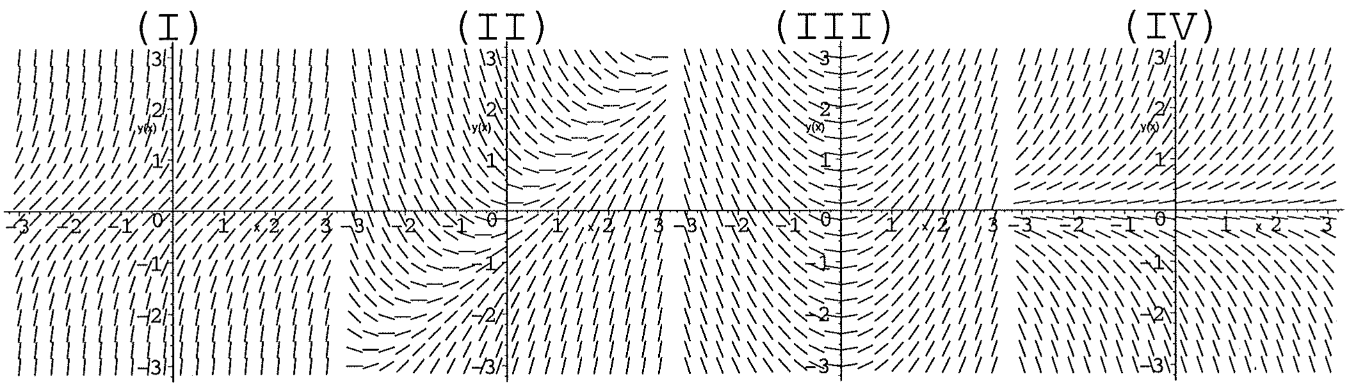
1. Solve the IVP

$$\frac{dy}{dx} + xy^2 = 0, y(1) = 1$$

2. Find the general solution to the differential equation $y'' + 6y' + 8y = 0$.

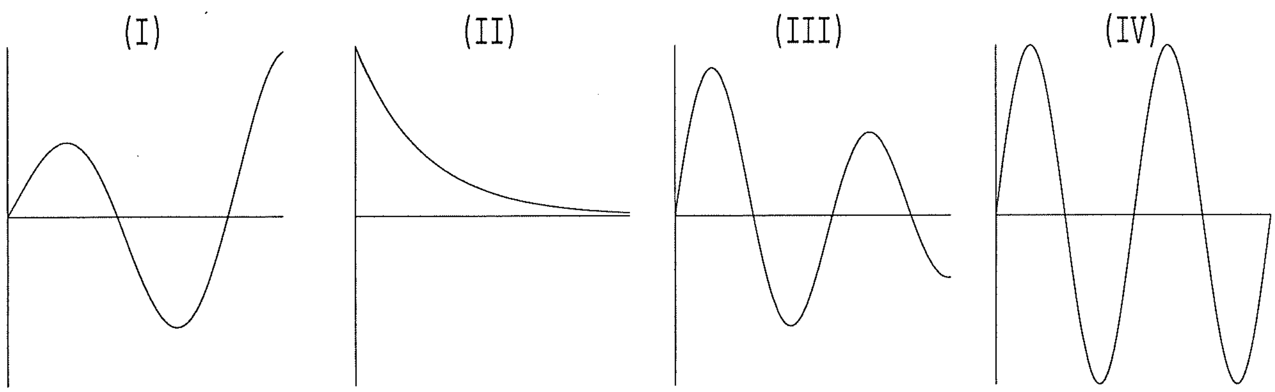
3. Find all values of r so that $y = e^{rt}$ is a solution to $y''' - 9y' = 0$?

4. Match the slope fields below with the differential equations $y' = 1 + y^2, y' = y, y' = x$ and $y' = x - y$.



5. Find the range of values on b which will make the general solution to the ODE $s'' + bs' + 5s$, overdamped? underdamped? and critically damped?

6. Match the graphs of the solutions below with the differential equations $y'' + 4y = 0, y'' - 4y = 0, y'' - 0.2y' + 1.01y = 0$ and $y'' + 0.2y' + 1.01y = 0$.

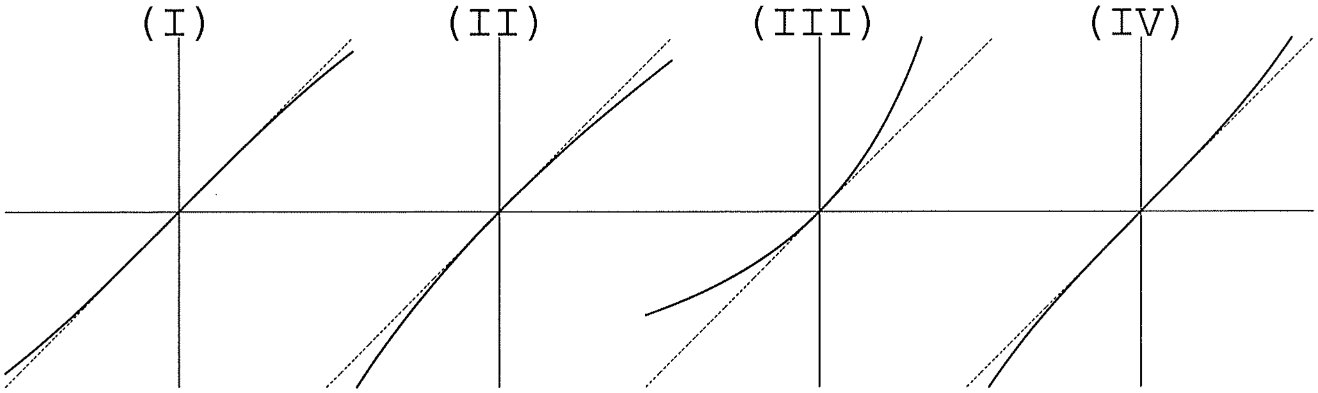


7. In the project you used a geometric series to estimate the error of truncating a power series to only N terms. Repeat this process to find an estimate of the error of the special case of using

$$\sum_{n=0}^{10} \frac{5^n}{n!}$$

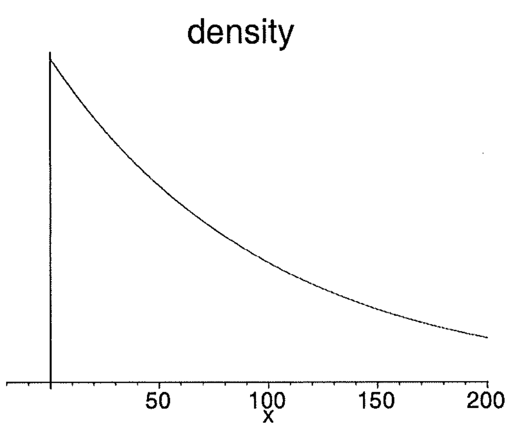
instead of the exact $e^5 = \sum_{n=0}^{\infty} 5^n/n!$. Be sure to explicitly give a and r and how you got them.

8. Find the first two non-zero terms of the Taylor series for the functions $\sin x$, $\tan x$, $x/(1-x)$ and $2\sqrt{1+x}-2$ and then match the functions to the graphs below (the thin dotted line is $y = x$).



9. Find exact values for the sum of the following series.

$$A = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$$
$$B = 1 + 0.1 + \frac{(0.1)^2}{2!} + \frac{(0.1)^3}{3!} + \dots$$
$$C = \frac{42}{100} + \frac{42}{10000} + \frac{42}{1000000} + \frac{42}{100000000} + \dots$$
$$D = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$$
$$E = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$$



10. The failure rate of a electronic device often has a decaying exponential probability density $\rho(x) = \lambda e^{-\lambda x}$ for $x \geq 0$ and $\rho(x) = 0$ for $x < 0$ (see above). For this problem, let $\lambda = 1/100$ and let x be time in seconds.

a. Find the percent of failures in the first 3 seconds.

b. Find the mean for this density.

c. A electronic supply firm wants to "burn in" each device T seconds before selling so that 99% of the failures happen during the "burn in" period. Find T.