MAC 2312 Calculus 2 **Test 1** 20 Sep 2001

Show **ALL** work for credit, correct answers are worthless without showing the process used to get them; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

1. Show all work needed to evaluate

$$\int 10^{1-x} dx$$

2. Show all work needed to evaluate

$$\int_1^4 \frac{\cos\sqrt{y}}{\sqrt{y}}\, dy$$

3. Show all work needed to evaluate

$$\int \frac{(t+2)^2}{t^3}\, dt$$

4. Show all work needed to compute the value of both the integrals below assuming $k > 0$.

$$\int_0^8 w^{-1/3}\, dw$$

$$\int_0^\infty e^{-kw}\, dw$$

5. Use the comparison test to show that both the given integrals converge.

$$\int_5^\infty \frac{1}{\theta^4+1}\, d\theta$$

$$\int_5^\infty \frac{1}{\theta^4-1}\, d\theta$$

6. Time and time again.
   a. Suppose a certain computer takes two seconds to compute a certain definite integral accurate to 4 digits to the right of the decimal point, using the left rectangle rule. How long (in years) will it take to get 12 digits correct using the left rectangle rule?
   b. Repeat part (a) but this time assume that the trapezoidal rule is being used throughout. Answer in "reasonable" units of time.

7. Show all work needed to evaluate

(1pt) $$\int \frac{1}{\sqrt{x}} dx$$

(3pt) $$\int \frac{1}{\sqrt{x+1}} dx$$

(6pt) $$\int \frac{1}{\sqrt{x}+1} dx$$

There is more test on the otherside

Welcome to side two

8. A limited amount of Maple, and a Mapleless limit.
 A. Write a correct maple expression for the following.

$$(3x^{-1} - \pi)(yz + \frac{1}{2a})^{w+5}$$

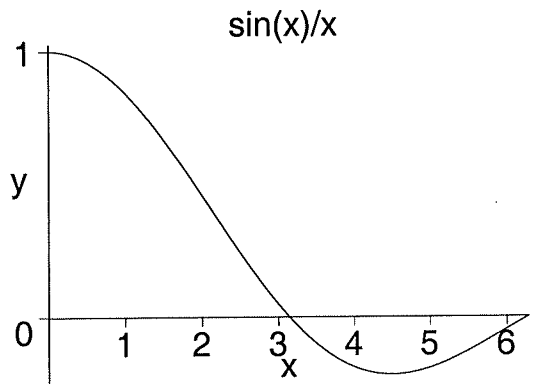
 B. Find the limit, justifying any use of L'Hopital's rule.

$$lim_{x \to \infty} \frac{\ln x}{x^2}$$

9. The graph of the function $f(x) = \sin x / x$ is given below. Consider

$$\int_{0.5}^{1} f(x)\, dx$$

 A. Arrange the approximations $LEFT(n), RIGHT(n), MID(n), TRAP(n)$ and the "true value" of the integral in incresing order.
 B. Use your trusty TI-89 to compute $TRAP(100)$ for this integral, report all the digits it gives.



10. Suppose for a certain definite integral that $TRAP(10) = 4.6893$ and $TRAP(50) = 4.6966$. Estimate the actual error for $TRAP(10)$ and the actual vaule of the integral by assuming that the error is reduced by a factor of roughly 25 in going from $TRAP(10)$ to $TRAP(50)$.

MAC 2312 Calculus 2 **Test 2** 25 Oct 2001

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

1. A one meter rod has density function $\rho(x) = 30 - 12x kg/m$ where x is the distance from the left end. Find the mass (in kg) and the center of mass (in m) of the rod.

2. Find the radius of convergence for the series below.

$$\sum_{n=0}^{\infty} \frac{2^n x^n}{n!}$$

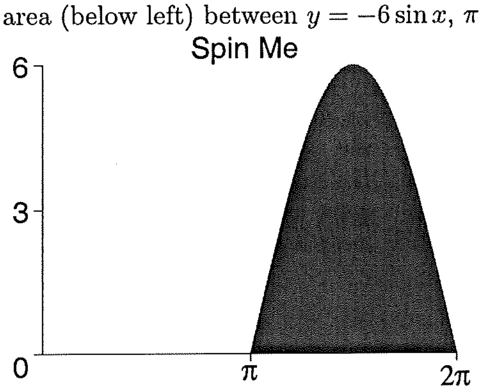
3. Find the radius of convergence for the series below.

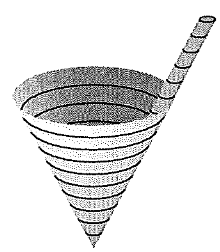
$$\sum_{n=0}^{\infty} \frac{(2n)! x^n}{(n!)^2}$$

4. Suppose that the power series $\sum_{n=0}^{\infty} C_n x^n$ converges for $x = -4$ and diverges when $x = 7$. Which of the following are true, false or not possible to determine? **Give REASONS** for your answers.

a. The power series converges when $x = 10$.
b. The power series converges when $x = 3$.
c. The power series diverges when $x = 1$.
d. The power series diverges when $x = 6$.

5. Write but do **NOT** evaluate integrals which will compute the volume of the figure obtained by rotating the area (below left) between $y = -6 \sin x$, $\pi \leq x \leq 2\pi$ and the x-axis about A. The x-axis. B. The y-axis.





6. Find the work (in N-m (Newton-meters)) done pumping all the water out of a full tank (above right) using a straw of whose top is at 5 meters. The shape of the tank is a cone that has height 3 meters and diameter 3 meters at the top. The density of water, ρ, is $1000 kg/m^3$ and g, acceleration due to gravity, is $9.8 m/s^2$.

7. Write and evaluate the integral which yields the arclength of the one quarter of the unit circle $x^2 + y^2 = 1$ in the first quadrant.

8. Consider the IVP

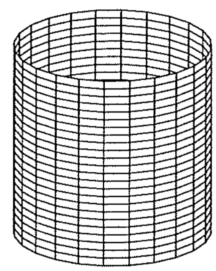
$$y' = 5 - y, y(0) = 1$$

(a) Use Euler's method by hand with two steps to estimate $y(1)$.
(b) Sketch the slope field for this ODE in the first quadrant, and use it to decide if your estimate is an over- or underestimate.
(c) Use Euler's method via your calculator to estimate $y(1)$ with ten steps.
(d) Use Euler's method via your calculator to estimate $y(1)$ with twenty steps.

9. A cup of java (below left) is made with boiling water and stands in a room where the temperature is $20°C$.
(a) If $T(t)$ is the temperature of the coffee at time t, explain what the ODE

$$\frac{dT}{dt} = -k(T - 20)$$

says in everyday terms. What is the sign of k?
(b) Solve this ODE. If the coffee cools to $90°C$ in 2 minutes, how long will it take to cool to $60°C$?



10. Derive but do **NOT** solve the differential equations below.
A $30m$ tall upright cylinderical tank (above right) has a circular base with area $100m^2$ and initially it has $1000m^3$ of fresh water (so initially the height of the water is $10m$). Into this large tank water flows a $3m^3/minute$ salt water solution containing 20 kilograms of salt per m^3. The solution is kept uniform in the tank by stirring. and the mixed water flows out according to the rules given below.

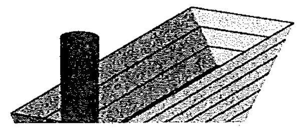
Case I: Assume the mixture is also leaving the tank at the same $3m^3/minute$ rate so that the $1000m^3$ volume remains constant.
(a) Derive a differential equation and initial values for Q the quantity of the salt in the tank (in kg).

Case II: Assume the water mixture now flows out at a rate proportional to the square root of the height $h(t)$ of the water instead of the constant rate in Case I.
(b) Derive a differential equation and initial values for h the height of the water in the tank. (Remember there is water flowing in as well as water flowing out.)
(c) Derive a differential equation and initial values for Q the quantity of the salt in the tank (in kg) using this second outflow assumption. You can use the $h = h(t)$ from part (b) in your equation.

Name ____________________

MAC 2312 Calculus 2 **Test 2 Annex** 1 Nov 2001

1. Find the work (in N-m (Newton-meters)) done pumping all the water out of a full tank (below right) using a pipe of whose top is at 5 meters above the bottom of the tank. The shape of the tank is a trianglar prism that has height 3 meters, width 3 meters at the top and is 10 meters long. The density of water, ρ, is $1000 kg/m^3$ and g, acceleration due to gravity, is $9.8 m/s^2$.



2. You can assume the two series below have radius of converence of one.

$$A = \sum_{n=1}^{\infty} \frac{x^{n+1}}{n(n+1)} \qquad B = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$$

Your job is to check the endpoints x=1 and x=-1 for convergence. Write down the resulting (non-power) series at each endpoint, write down whether each endpoint series converges or diverges and finally give the interval of convergence of each power series.

Name ____________________

MAC 2312 Calculus 2 **Test 3 Bonus** 4 Dec 2001

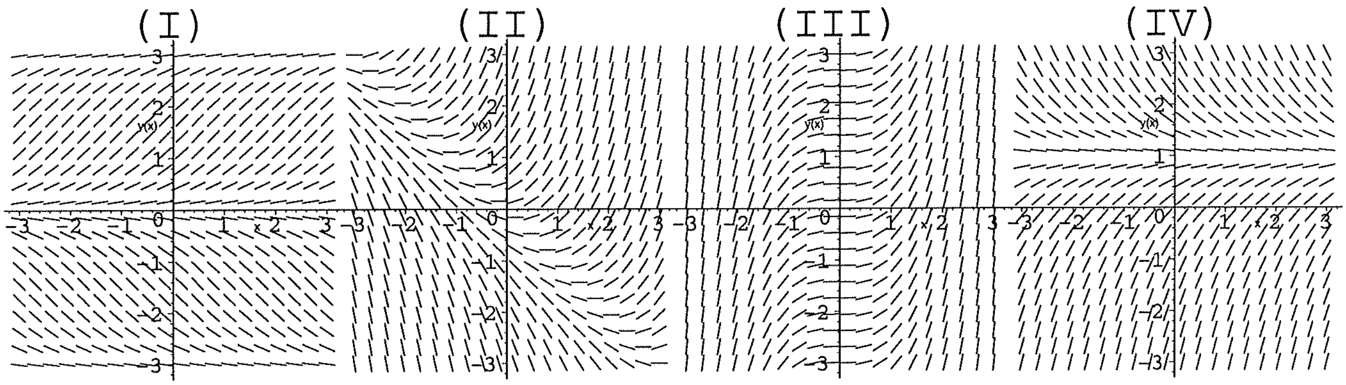
1. Solve the initial value problem

$$\frac{d\omega}{d\theta} = \theta\omega^2 \sin\theta^2, \qquad \omega(0) = 1$$

MAC 2312 Calculus 2 **Test 3** 29 Nov 2001

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

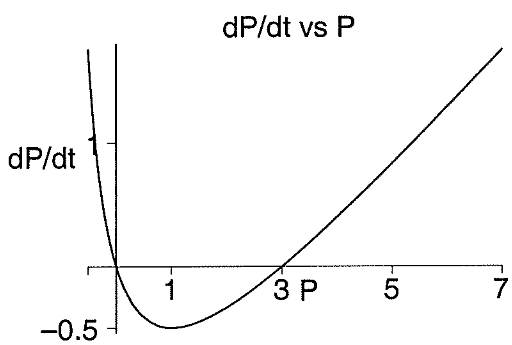
1. Match the slope fields below with the differential equations $y' = x^2, y' = \sin y, y' = 1 - y$ and $y' = x + y$.

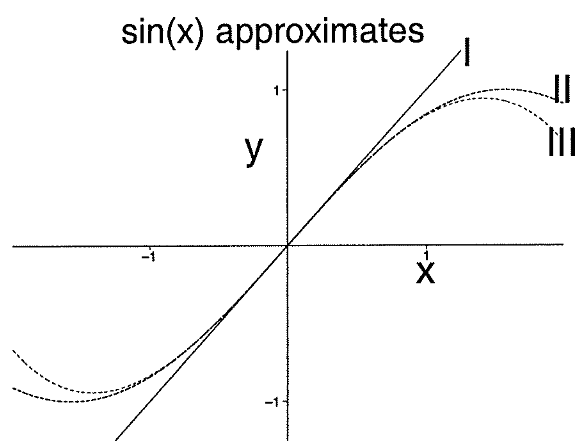


2. Find all values of r so that $y = x^r$ is a solution to $x^2y'' + 2xy' - 6y = 0$.

3. Find the solution of $y' = -y/x,\ y(1) = -2$. Sketch the graph of your solution.

4. For the differential equation $\frac{dP}{dt} = f(P)$, the graph of $f(P)$ or $\frac{dP}{dt}$ versus P is given below (left).
   a. Sketch a graph of the slope field for this differential equation.
   b. Find both equilibrium solutions, and label them as stable or unstable.
   c. On your slope field, find and sketch a solution with an inflection point, and label your inflection point with its coordinates.





5. Approximating $\sin x$ by x and $x - x^3/3!$.
   a. The graph to the above (right) graphs these three functions, identify which is which. [Hint: What is the next term in the Taylor series?]
   b. Use $\sin x \approx x - x^3/3!$ to show

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

.

   c. Use $\sin x \approx x - x^3/3!$ to estimate (to five significant digits) the relative error $= (\sin x - x)/\sin x$ of using x to approximate $\sin x$ for the angle x in radiams that corresponds to $15°$. [So you are using the better approximation to gauge the error in the simpler approximation.]

6. Consider the IVP

$$y' = 1 + y^2, y(0) = 1$$

(a) Use Euler's method by hand with two steps to estimate $y(1)$.
(b) Sketch the slope field for this differential equation in the first quadrant, and use it to decide if your estimate is an over- or underestimate.
(c) Use Euler's method via your calculator to estimate $y(1)$ with ten steps.

7 & 8. These problems are about the Taylor series for the functions f and g given below.

$$f(x) = \sum_{n=1}^{\infty} nx^n = x + 2x^2 + 3x^3 + 4x^4 + \ldots$$

$$g(x) = \sum_{n=1}^{\infty} 2^{n-1}x^n = x + 2x^2 + 4x^3 + 8x^4 + \ldots$$

a. Is f or g larger for small positive x and why?
b. Find $g'''(0)$.
c. Using substitution, find the Taylor series for $f(2u)$.
d. Find the Taylor series for $g'(x)$.
e. Show the multiplication needed to get the first 4 non-zero terms of the Taylor series for fg.

9. A Calculus class at a party school brings an ice cream cake to a 7:30 final. The cake is frozen ($40°F$) too hard to eat right away. Besides the class is eager to take the final. Two hours later the cake is at $50°F$ and is eaten immediately. The classroom is at a constant $70°F$.

a. Assuming the temperature, T, of the cake obeys Newton's Law of Cooling, write a differential equation for T.
b. Solve the differential equation to estimate the time the cake was taken out of a $30°F$ freezer.

10. Santa is making a list. He is adding names at the rate of 1 million names a day and 99 % of the new names are "nice". At the same time 1 million names a day are randomly selected to fall off the list. (The list always has the same number of names.) The list starts with 1 billion people, 95 % of whom are nice. Derive a differential equation and initial conditions for N the number of "nice" people on the list. Do **NOT** solve your IVP. [Hint: It is like salt in water.] Be sure to check your list twice.