

In 1-4 find the derivative of the given function.

1. $g(x) = x^{10} - 9x^{-1} + \frac{x^{7/8}}{7} + 2x^2 + \pi$

2. $H(x) = \frac{(x+x^{-1})^2}{(x+1)^2}$

3. $(x^2 + x^{1/2} + 3)^{1/2} (x^3 + x^{-3})^5 = f(x)$

4. $((x^2+1)^3 + x^3)^5 = S(x)$

In 5-7 find the limits.

5. $\lim_{x \rightarrow 10} \sqrt{x^2+1}$

6. $\lim_{x \rightarrow 0} \frac{x^5 + 3x^3 + x}{x^7 - 7x}$

7. $\lim_{x \rightarrow 1^+} \frac{\sqrt{x+3} - 2}{x-1}$

8. Use the three step rule to find $f'(x)$ for the function $f(x) = 1/x$.

9. Give the ϵ - δ definition for $\lim_{x \rightarrow a} f(x) = L$.

Find a δ that works for $\epsilon = 0.1$ $a = L = 1$ and $f(x) = \sqrt{x}$.

10 Let $f(x) = \begin{cases} 3x-6 & -\infty < x < 10 \\ x & -10 \leq x < 10 \\ x^2-9x & 10 \leq x < \infty \end{cases}$

Where is $f(x)$ continuous?

MATH 151 TEST IIIA SHOW ALL WORK BE NEAT
USE ONE SIDE OF EACH PAGE ONLY

1. Find an antiderivative to $\frac{x^{100}}{101} + x^7 + 3x^2 + \frac{x}{2} + 7$
2. Find $\frac{dy}{dx}$ implicitly if $x^2y^2 + y + xy = 7\pi$
3. Find $df(x, h)$ for $f(x) = (x + x^{-1})^{-1}$
4. Approximate $\sqrt{4.2}$
5. Find the area under $f(x) = 1 - x^2$ from $a = -1$ to $b = 1$
6. $W = \sqrt{1 - t^2}$ $t = s^3 + 3s^2 + \frac{1}{s}$ Find $\frac{dW}{ds}$
7. Find $\lim_{x \rightarrow -\infty} \frac{x^{10} + 17x^7 + 5}{10x^{10} + x + 1}$
8. Find $\lim_{n \rightarrow \infty} a_n$ where $a_n = \sqrt{n^2 + 1} - n$
9. A superball is dropped from 144 ft tower. How long does it take to hit the ground?
Hint: $a(t) = -32$ $v(0) = 0$ $s(0) = 144$.
10. A stone is thrown into a pond and makes a circular wave radiating in all directions with the radius increasing at 7 ft/sec. How fast is the Area enclosed by the wave increasing when the radius is 100 ft.?

MEMO

To: Members of the Mathematics Coffee Club

- J. Andrews
- S. Belletot
- S. Blunsack
- J. Davis
- D. Goodner
- H. C. Giffith
- C. Jam
- K. Yanosko
- P. Novinger
- B. Batson
- C. Lacher
- P. Jenkins
- R. McArthur
- R. Williams
- A. Selman
- P. Smith
- J. Snover

From: Pam Jenkins

The coffee pot belonging to the Coffee Club is broken. We have had to replace this pot yearly for the past couple of years. Should we simply continue replace the pot as usual (it costs \$12.00), or should we look for an alternate solution? Please let me know your opinions.

March 13, 1975

MATH 151 TEST III SHOW ALL WORK BE NEAT
USE ONE SIDE OF EACH PAGE ONLY

1 Find an antiderivative to $x^{100} + 7x^7 + x^2 + x + 3$

2. Find $\frac{dy}{dx}$ implicitly given $x^2y^2 + xy = 7$

3 Find $\lim_{x \rightarrow +\infty} \frac{x^5 - x^3 + 7}{5x^5 + x^4 + 2x}$

4 Find $df(x, h)$ for $f(x) = \frac{x^2}{x^2+1}$ $\frac{2x(x^2+1) - x^2(2x)}{(x^2+1)^2}$

5 Approximate $\sqrt{1.01}$

6 Find the area under $f(x) = 1 - x^2$ from $a = -1$ to $b = 1$

7 $V = \pi h^3$; $h = \sqrt{1 + s^2}$ Find $\frac{dV}{ds}$.

8 Given $a(t) = 1 - 2t$ $v(0) = 0$ and $s(0) = 1$
Find $v(t)$ and $s(t)$.

9 Find $\lim_{n \rightarrow \infty} a_n$ where $a_n = \frac{\sqrt{n^2+1} - n}{n}$

10. A snowball is melting and losing volume at a rate of $2 \text{ in}^3/\text{min}$. Find the rate of decrease of the radius when the radius is 3 in.

HINT: Volume of sphere is $\frac{4}{3}\pi$ times the radius cubed.

MATH 151 TEST IIA SHOW ALL WORK; BE NEAT;
USE ONE SIDE OF EACH PAGE.

- 1) $s(t) = \frac{t+1}{t-1}$ FIND $v(t)$ & $a(t)$
- 2) Find the equation of the tangent line to $g(x) = \frac{1}{x^2} + 1$ at $x = 2$.
- 3) GIVEN $F'(x_0) = 0$ and $F''(x_0) < 0$, what can you say about $F(x)$ NEAR x_0 ? SKETCH
- 4) GIVEN $F'(x_0) = F''(x_0) = 0$ AND $F''(x) < 0$ FOR $x < x_0$ AND $F''(x) < 0$ FOR $x > x_0$. WHAT CAN YOU SAY ABOUT $F(x_0)$ SKETCH.
- 5) GIVEN $F'(x) = 0$ everywhere what is $F(2) - F(1) = ?$
- 6) $F'(x) = (x-1)(x-2)(x-3)$ WHERE IS $F(x)$ INC, DEC? WHERE THE RELATIVE MIN'S & MAX'S,
- 7) FIND THE MIN AND MAX VALUES OF $g(x) = x^{-2} + x^2$ ON $[\frac{1}{10}, 10]$
- 8) FIND THE MIN AND MAX VALUES OF $f(x) = \sqrt{x^3 + 3x}$ ON $[0, 2]$
- 9) $F(x) = (x+4)(x-2)^2$
 $F'(x) = 3(x+2)(x-2)$
 $F''(x) = 6x$ } FIND WHERE F, F', F'' ARE $= 0, > 0, < 0$
FIND WHERE F IS INC, DEC, CONCAVE UP OR DOWN, REL MIN'S & MAX'S AND PTS OF INFLECTION. SKETCH
- 10) Find the point on the line $2x + y = 10$ nearest the origin.

1. If $f(x) = x^{72} + \frac{x^{18}}{18} + 7x^7 + \frac{x^3}{2} + \pi$, then $f'(x) = ?$

2. If $g(x) = \frac{x+1}{(x-1)^{1/2}}$, what is $g'(x) = ?$

3. If $F(y) = (y^2 + y + 1)^2 (y^{3/2} + y^{1/2} + y^{-1/2})^3$, then $F'(y) = ?$

4. If $f(z) = \frac{z + \sqrt{z^2 + 1}}{1 + z^2}$, what is $f'(z) = ?$

5. If $f(x) = (x^2 + (x^{-2} + x^2)^{1/2})^{10027}$, what is $f'(x) = ?$

6. Implicitly find y' ~~of~~ given $x^4 + x^2 y^2 + y^4 = 1$.

7. For what values of c in (a, b) does

$$\frac{f(b) - f(a)}{b - a} = f'(c) \text{ for } f(x) = x^3, a = 0, b = 1?$$

8. Find the equation of the tangent line of $f(x) = x^3 - 7$ at $x = 2$.

$$s(t) = t^4 + \frac{4}{3}t^3 - 12t^2$$

9. Let $s(t)$ be the position, at time t , ~~of~~ of a moving particle on a line positive to right negative to left.

Find the velocity $v(t)$ and the acceleration $a(t)$.

Find the values of t for which the particle is at rest when it is moving to the left and when it is moving to the right? [Be Explicit].

10. For $x^4 + y^4 = 10001$, find the equation of the normal line at the point $(10, -1)$

directions:

- (1) Show All Work
- (2) Be Neat
- (3) Use one side of each page only
- (4) You do not need to use the limit theorems to find any limits
- (5) Each Problem is worth 10pts, you may do them in any order.
- (6) Good luck

$$1. \lim_{x \rightarrow 4} (x^3 - 5x^2 + x + 7) = ?$$

$$2. \lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{x^3 - 1} = ?$$

$$3. \lim_{x \rightarrow 2} \frac{\sqrt{x+2} - 2}{x-2} = ?$$

$$4. \lim_{x \rightarrow +\infty} \frac{2x^5 - x^3 + 1}{4 - x^2 + 4x^5} = ?$$

$$5. \lim_{x \rightarrow 1} \frac{x}{(x-1)^2} = ?$$

$$6. \lim_{h \rightarrow 0^+} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} = ?$$

$$7. \lim_{x \rightarrow 3^-} \frac{|x-3|}{x-3} = ?$$

8. Give the ϵ - δ definition of $\lim_{x \rightarrow a} f(x) = L$.

9. Find which δ (or δ 's) will work in the definition with $f(x) = \sqrt{2x}$, $a=2$, $L=2$, and $\epsilon = .5$.

10. For $f(x)$ define below, tell where $f(x)$ is continuous and where it is discontinuous. Defend your answers.

$$f(x) = \begin{cases} 2 - x^2 & -\infty < x \leq 0 \\ 2 - x & 0 < x < 1 \\ x^2 + 2 & 1 \leq x < +\infty \end{cases}$$

MATH 151 FINAL PAGE ONE SHOW ALL WORK; BE NEAT
USE ONE SIDE OF EACH PAGE ONLY

IN 1-4 FIND THE LIMITS

1. $\lim_{x \rightarrow -1} (x^4 + x^3 - 7x^2 + 15)$

2. $\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$

3. $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x^2 - 2x - 3}$

4. $\lim_{x \rightarrow +\infty} \frac{3x^5 - x^3 + 100}{x^2 + 13x - x^5}$

IN 5-8 FIND $f'(x)$ if $f(x)$ is

5. $x^{101} + 27x^{27} + x^{-3} + \frac{x^{17}}{17} + \pi^2 \sqrt{2}$

6. $(x + x^{-1})^{10} (x^2 + x^{-2})^{-1}$

7. $\frac{x + \sqrt{x}}{x + 1}$

8. $(x + (x^2 + 1)^{20})^{-3}$

9. Find an anti derivative to

$$f(x) = 3x^2 + x^{17} - x^{-2} + 7 + x$$

10. Implicitly find $\frac{dy}{dx}$ if $x^4 + x^2 y^2 + y^4 = 1$

11. Use the differential to approximate $\frac{1}{\sqrt{.99}}$

12. Find the equation of the tangent line to $f(x) = x^2 + \frac{25}{x}$ at $x = 5$.

13. Find the minimum value and maximum value of $f(x) = 2x^3 - 9x^2 + 12x + 1$ on $[0, 3]$.

14. Find the area between: $f(x) = x^3$, the x -axis, $x = 1$ and $x = 2$; i.e. $\int_1^2 x^3 dx$.

15. If the velocity is given by $v(t) = t^3 - 7$, find the acceleration $a(t)$ and $s(t)$ if $s(2) = 3$.

16. FIND ALL RELATIVE MAXIMUMS, RELATIVE MINIMUMS AND POINTS OF INFLECTIONS OF $f(x) = x^3 + x^2$.

17. Where is $f(x) > 0$ & $f(x) < 0$ if

$$f(x) = -7(x-1)(x-2)(x-3)^2(4-x)(x^2+1)^2$$

18. GRAPH the continuous function $f(x)$ given

$$f(1) = f(5) = 2, \quad f(3) = -2, \quad \lim_{x \rightarrow -\infty} f(x) = 0$$

$$f'(x) = 0 \quad \text{at } x=1, x=3, x=5 \text{ and } 5 < x < +\infty.$$

$$f'(x) > 0 \quad \text{at } -\infty < x < 1, 3 < x < 5$$

$$f'(x) < 0 \quad \text{at } 1 < x < 3$$

$$f''(x) = 0 \quad \text{at } x=0, x=2, x=4 \text{ and } 5 \leq x < +\infty.$$

$$f''(x) > 0 \quad \text{at } -\infty < x < 0, 2 < x < 4.$$

$$f''(x) < 0 \quad \text{at } 0 < x < 2, 4 < x < 5.$$

19. A snowball in the shape of a sphere is melting at a rate (loss of volume) of $10 \text{ in}^3/\text{min}$. How fast is the radius decreasing when the radius is 10 in. Hint: Volume of sphere is $\frac{4}{3}\pi$ times radius cubed.

20. Find the point on the curve $y = \sqrt{x}$ which is closest to the point $(9, 0)$.