The Bellenot's
MAC 3311- 09

Calculus I

Spring, 1990
SYLLABUS

The Good Doctor's OFFICE is currently 218 Love but will soon be 002B(?) Love
His OFFICE HOURS are by appointment. (The Good Doctor is around most afternoons.)

ELGIBILITY. You must have the course prerequisites listed below, and must never have completed with a grade of C- or better a course for which MAC 3311 is a (stated or implied) prerequisite. Students with prior credit in college calculus are required to reduce the credit for MAC 3311 accordingly. It is the student's responsibility to check and prove elgibility.

PREREQUISITES. You must have passed MAC 1140 (College Algebra) and MAC 1113 (Trigonometry) (or MAC 2140 and MAC 1114 at TCC) with a grade of C- or better, or have appropriate transfer credit. Placement in AMP Group 1 or 1H (or 2 if you are currently taking trigonometry) is also considered to satisfy the prerequisite.

TEXT. Calculus with Analytic Geometry, Seventh Edition, Thomas and Finney.
COURSE CONTENT. Chapters 1-5.

ATTENDANCE/HOMEWORK. Mathematics is not a spectator sport. A student who does not turn in her/his homework at the start of class is deemed ABSENT. A student with more than 5 absences will automatically FAIL. No late homework is accepted.

EXAMS. Final at 12:30-2:30 Thurs 26 Apr. Tentative unit test dates: 29 Jan, 19 Feb, 12 Mar and 16 Apr. Every Tuesday another "Take Home Test Problem" is due. No makeup exams will be given, however the lowest of the in-class-unit-test grades will be dropped. No late "take home test problems" are accepted (but see below). All in-class tests and the final are "closed book".

GRADING. The final exam will be weighted as two unit tests in calculating final averages. The best ten of the thirteen take home test problems will be weighted as a unit test. Letter grades for students with good attendance records will be based on numerical grades in the usual way (A: 90-100; B: 80-89; C: 70-79; D: 60-69; F: 0-59).

HELP CENTER. The help center will be held in 110 MCH (old EDU). Sessions will begin on January 16th and will be held during the following hours: Monday - Thursday 12:30 - 9:00 p.m., Friday 12:30 - 4:00 p.m., and Sunday 2:00 - 5:00 p.m.

TAKE HOME TEST PROBLEMS. Reminber these problems are tests and the MINIMAL punishment for cheating is an automatic FAIL in the course. The list of the problems follows. You may use both your text and your notes to help you on the Take Home TP's.
TP's are required to be:

1. In INK.
2. On 8-1/2 by 11 paper.
3. One side of each page only.
4. Multiple pages (if any) stapled or paper clipped together.

TP's are graded on

1. The correct answer(s).
2. Your ability to explain what you are doing (including your English).
3. Scores of particularly overly uneconomical solutions will be decreased.

Spring 1990 Calculcus I
Take Home Test Problems:

1. (Due 16 Jan) Find an equation of the tangent line to $x^2 + y^2 = 9$ which passes through the point (0,5) and has negative slope.

2. (Due 23 Jan) (Numerical Problem) Angle A of a triangle is 1.314 RADIANS and the lengths of the two sides incident to A are 2.043 FEET and 39.37 INCHES. Use the law of sines (Appendix 4) to find the angle opposite the side which is 2.043 feet and use the law of cosines to find the length of the third side.

3. (Due 30 Jan) Prove by induction (Appendix A2 or p272):

$$\sum_{i=0}^{n} \frac{(2+i)(1+i)}{2!} = \frac{(3+n)(2+n)(1+n)}{3!}$$

4. (Due 6 Feb) (Numerical Problem) Angle A of a triangle is 1.169 radians and the side opposite angle A is 1.013 meters. One of the sides incident to A is 1.223 meters. There are two possible lengths for the third side, find both of them and the resulting angles opposite this third side.

5. (Due 13 Feb) (#50 p117) If 3 normals can be drawn from A(a,0) to the curve $y^2 = x$, show a must be greater than 1/2. One normal is always the x-axis. Find the value of a for which the other two are perpendicular.

6. (Due 20 Feb) Prove by induction:

$$\sum_{i=0}^{n} (3+i)(2+i)(1+i) = \frac{(4+n)(3+n)(2+n)(1+n)}{4}$$

7. (Due 27 Feb) Consider the lemniscate $(x^2 + y^2)^2 = x^2 - y^2$, whose graph looks like "∞". Find the points on the graph at which the tangents are horizontal. (*Hint:* There are four such points.)

8. (Due 6 Mar) During the sixteenth and seventeenth centuries, wine merchants in Linz, Austria, calculated the price of a barrel of wine by first inserting a measuring rod as far as posible into a taphole located in the middle of the side of the barrel and then charging the customer according to the length L of the rod. Assumming that the barrel is right circular cylinder, find the dimensions of the barrel of largest volume for a given length L of the rod.

9. (Due 13 Mar) Water is released from a conical tank (with vertex pointed downward) with height 50 meters and radius 30 meters and falls into a rectangular tank whose base has an area of 400 square meters. The rate of release is controlled so that when the height of the water in the conical tank is x meters, the height is decreasing at the rate of 50 - x meters per minute. How fast is the water level in the rectangular tank rising when the height of the water in the conical tank is 10 meters? (*Hint:* The total amount of water in the two tanks is constant.)

10. (Due 27 Mar) Three sides of a trapezoid are of equal length L, and no two are parallel. Find the length of the fourth side that gives the trapezoid maximum area.

11. (Due 3 Apr) A street light 16 feet high casts a shadow on the ground from a ball that is dropped from a height of 16 feet but 15 feet from the light. How fast is the shadow moving along the ground when the ball is 5 feet from the ground. (*Hint:* The distance s from the ball to the ground t seconds after the release is given by the equation $s = 16 - 16t^2$.)

12. (Due 10 Apr) Find the length of the largest thin, rigid pipe that can be carried from one 10-foot-wide corridor to a similar corridor at right angles to the the first. Assume that the pipe has negligible diameter. (*Hint:* Find the length of the shortest line that touches the inside corner of the hallways and extends to both walls.)

13. (Due 17 Apr) Find the volume of the solid with the given information about its cross sections. The base of the solid is an isosceles right triangle whose legs L_1 and L_2 are 4 units long. Any cross section perpendicular both to the base and L_2 is semicircular.

CAL I TEST 3 12 MAR 90 by ____________

Show ALL Work. Be careful with equal signs. Read the problems CAREFULLY

1. $\lim_{x \to 0^+} \frac{\sin 2x}{\sin 3x} =$

2. $\int 6x^5 + \frac{x^7}{7} + \sqrt{2x} + \frac{1}{x^3} + \pi \, dx =$

3. $\int \frac{t^2 dt}{\sqrt{3t^3 + 20}} =$

4. Find $f(t)$ if $f''(t) = 5(t-7)^3$, $f'(9) = 36$ and $f(7) = 10$

5. Find the equation of the tangent line to $y \cos 2x - x \sin 2y = \pi/4$ at $(\pi/6, \pi/2)$. Simplify.

6. For $f(x) = \sqrt{x-1}$ $a = 1$, $b = 3$ find all c's which will make the MVT (Mean Value Theorem) true.

7. Find the minimum and maximum VALUES of $f(x) = 2x^3 - 15x^2 + 24x$ on $[3, 5]$

8. Find the minimum and maximum VALUES of $g(z) = \frac{z}{1+z^2}$ on $(0, 3)$

9. A 13 foot ladder is leaning against a wall. The foot of the ladder is moving towards the wall at 3ft/sec. How fast is the top of the ladder rising when the top of the ladder is 12 ft above the ground? (Assume the wall and the ground are perpendicular.)

10. Find the dimensions of the rectangle of maximum area which can be constructed using 4 miles of fence for THREE of the four sides. (The fourth side isn't fenced.)

CAL I TEST 4 16APR90 by ____________________

Show ALL Work. Be careful with equal signs. Read the problems CAREFULLY.

1. $\int \tan t \sec t + \cos 2t + \sec^2 \frac{t}{3} + 2t \sin t^2 \, dt =$

2. $\int_{\pi/4}^{\pi/3} \cos^2 t \, dt =$

3. $\int_{-3}^{-2} (2x-3)\sqrt{x+3} \, dx =$

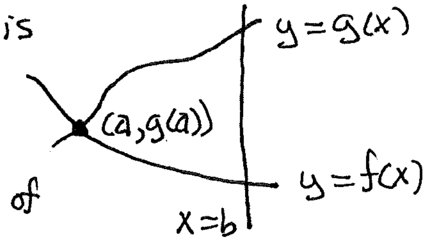
4. Find $F'(x)$, if $F(x) = \int_1^{2x} \sqrt{1+t^3} \, dt$.

5. Find the area between $y = 2x - x^2$ and the x-axis.

6. A thin rod has density function $\delta(x) = (4-x)x^2$ for $0 \leq x \leq 4$. Find the mass and center of mass of the rod.

7. The parametric equations to right are those of a spiral. Write an integral which is the arclength of this spiral. Simplify the integrand, but do NOT evaluate the $\int$.

$$\begin{cases} x = t\cos t \\ y = t\sin t \end{cases} \quad \pi \leq t \leq 5\pi$$



8. The region S is pictured to the right. It is the area between $y=f(x)$, $y=g(x)$ and $x=b$. For parts A & B write an Integral (but do NOT evaluate it) which is the volume of the solid of revolution given by rotating S about the given axis.

A. The x-axis

B. the y-axis

9. The line $15y-3x+18=0$ and the curve $x=y^4$ intersect at $(1,-1)$ and $(16,2)$. Find the area between the line and the curve.

10. The error estimate for the trapezoidal rule is $|E_T| \le \frac{b-a}{12}h^2M$, where M is the maximum value of $|f''|$ on $[a,b]$. Give the minimum number of subdivisions "n" which are need to approximate $\int_2^3 x^4\,dx$ with an error less than 10^{-4} using this error estimate.

Old Test 3 1* $\lim\limits_{x\to\infty} \dfrac{4-10x^2+3x^3}{5x^2-7x^3-2}$ 2.† $\int x^3+3\sqrt{x}+x^{-2}+3\pi^2\sin \pi x \, dx$

3. $\int t^2(3t^3+100)^{-200}\,dt$ 6† $\int \dfrac{3x+4}{\sqrt{x+1}}\,dx$

10. Find the dimensions of the rectangle of maximum area which can be inscribed in a semi-circle of radius R

Old Test 2 2. Find $f(t)$ given $f''(t)=60t^2+2$, $f'(3)=1000$, $f(-1)=70$

6. Find the min and max VALUES of $g(s)=s^2+s^{-2}$ on $[\tfrac{1}{4},2]$

7. Find the min and max VALUES of $(1+x^2)^{-1}$ on $(-10,20)$

10.* A floor light (height zero) is moving toward a 6ft man at 3ft/min. On the otherside of the man, 20ft away is an infinitely tall wall. How fast is the height of the man's shadow on the wall increasing when the light is 30ft from the wall.
(See also #10 on Old Test 1)

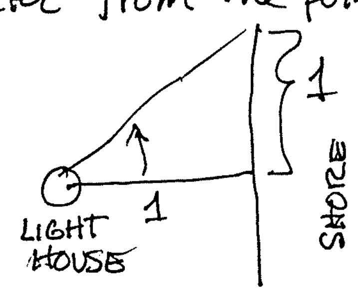
Old Test 4
7.† $\int \dfrac{4u+7}{(3u-2)^2}\,du$
← can't do this one yet

Old Test 5
7.† $\int \dfrac{3x+7}{(2x-6)^3}\,dx$

A. If $f(x)=x^{2/3}$ $a=0$, $b=1$ find all c's which expresses the MVT.

B. $\lim\limits_{x\to 0^+}\left(\dfrac{1}{x}-\dfrac{1}{\sqrt{x}}\right)$ C. $\lim\limits_{x\to 0}\dfrac{x(\cos x-1)}{\sin x - x}$

D. A light house is 1 kilometer from the shore. The light rotates at 2 radians/sec. How fast along the shore line is the light moving at a point on the shore line 1 kilometer from the point on the shore nearest the light house.



E. Find the point on the curve $f(x)=\sqrt{x}$ nearest the point $(1,0)$

F. Find the equation of the tangent line to $\tan x \sec y = \sqrt{6}$ at $(\pi/3, \pi/4)$. Simplify.

Last Prac. Test

1. $\frac{d}{dx}\int_{x^2}^{2} \sqrt{1+t^3}\,dt =$

2. $\int \sin^3 2t\,dt =$

3. $1+2+3+\ldots+998+999+1000 =$

4. $\int_1^9 \frac{x-3}{(2x+7)^{1/2}}\,dx =$

5. $\int_0^{\pi/2} \sqrt{1+\sin^2 x}\,\sin x\cos x\,dx =$

6. $\int \tan^2 3t\,dt =$

7. & 8 Are about the region S' which is between the curves $f(x)=x^2-4x+7$ and $g(x)=-x^2+4x+1$

7. Write an integral (but do NOT evaluate it) which is the volume of the solid obtained by rotating S' about the x-axis

8. Write an integral (but do NOT evaluate it) which is the volume of solid obtained by rotating S' about the y-axis.

9 & 10 Are about the region R which is between $y=4$, $y=x^2$ and $y=x^3/16$

9. without interchanging x & y, write an integral (or sum of integrals) with respect to dy which is the area of R. do NOT evaluate the $\int$(s).

10. without interchanging x & y, write an integral (or sum of integrals) with respect to dx which is the area of R. do NOT evaluate the $\int$(s).

11. Find the surface area when $y=x^2/2$ $0\le x\le 2$ is rotated about the y-axis.

12. For the function $f(x)$ to the right $f(x)=\begin{cases}-2 & -8\le x\le -3\\ 5 & -3<x<1\\ 1 & 1\le x\le 8\end{cases}$ find $\int_{-8}^{8} f(x)\,dx$ and both the approximations given by the Trapezoid rule with $n=4$ and Simpson rule with $n=4$.

13. A thin rod has density function $\delta(x)=x^3$ for $0\le x\le L$. find the mass and center of mass of the rod.

14. Find the point on the curve $y=\sqrt{x}$ nearest to the point $(9,0)$.

15. The error estimate for Simpson's rule is $|E_s|\le \frac{b-a}{180}h^4 M$ where M is the maximum value of $|f^{(4)}|$ on $[a,b]$. Give the minimum number of subdivisions "n" which are needed to approximate $\int_1^2 x^{-3}\,dx$ with an error of less than 10^{-4}?

16. The last take home test problem was a problem on one of the hour tests in Calculus I last time.