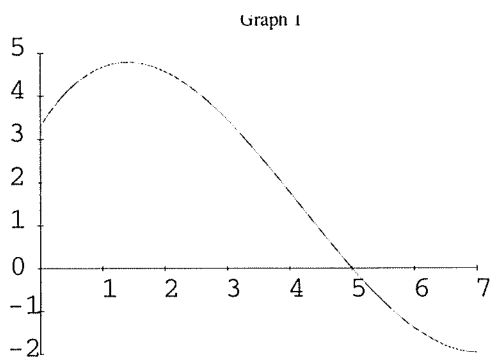
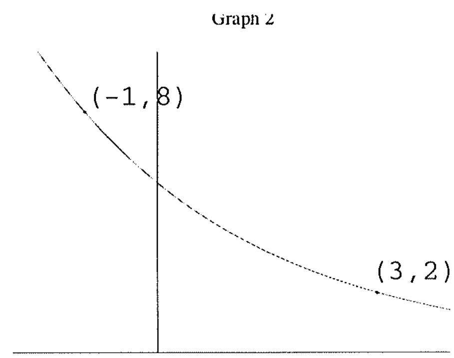
MAC 2311 Calculus 1 **Test 1** 28 Jan 1999

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only.

1. The entire graph of $y = f(x)$ is shown in Graph 1 below. Find
(a) The domain of $f(x)$.
(b) The range of $f(x)$.
(c) List all roots of $f(x)$.
(d) Is $f(x)$ concave up or concave down at $x = 6$?
(e) Is $f(x)$ concave up or concave down at $x = 3$?



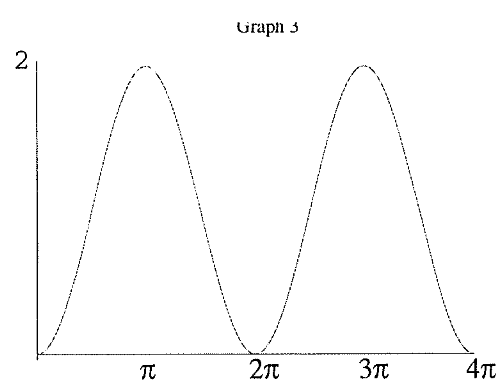


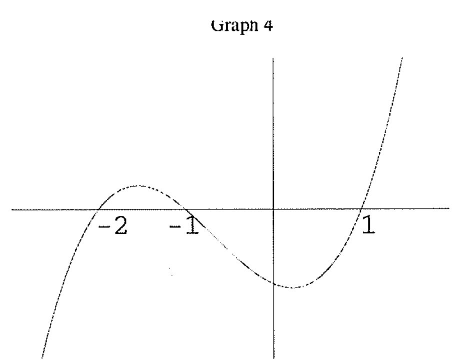
2. Find a possible equation involving a exponential for Graph 2 above. Write your equation in the $Q = Q_0 a^t$ format.

3. In each part determine which function has larger values as $x \to \infty$
(a) $x^{1/3}$ or $1000 \log x$.
(b) $1000 \cdot (1.01)^x$ or x^{101}.

4. Find the half-life of a radioactive substance that is reduced by 30% in 20 hours.

5. Find a possible formula for the Graph 3 below. Give the period and amplitude of your function.





6. Find a possible formula for the Graph 4 above. If necessary, modify your formula for the function $f(x)$ so it satisfies the additional condition that $f(0) = -1$.

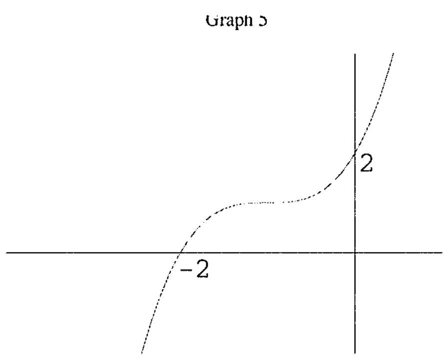
There is more test on the other side.

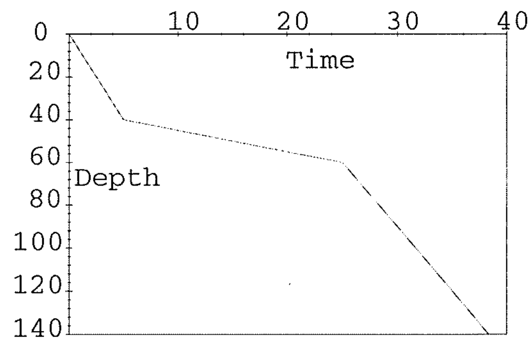
Welcome to test one side two.

7. The table below contains values for three different functions.
(a) Which (if any) for these functions are linear functions? For those functions which are linear, find the formula.
(b) Which (if any) of these functions are exponential functions? For those functions which are exponential, find the formula.

x	$f(x)$	$g(x)$	$h(x)$
-2	12	16	37
-1	17	24	34
0	20	36	31
1	21	54	28
2	18	81	25

8. By shifting the graph of $y = x^3$, find a cubic polynominal with the graph of Graph 5 below.





9. The graph to the above right plots $f(t)$. Here $f(t)$ is the depth in meters below the Atlantic Ocean floor where t million-year-old rocks can be found. The data is from core samples drilled by the research ship *Glomar Challenger*, drawn by Maple using a piecewise approximation to the data.
(a) Evaluate $f(15)$, and say what it means in practical terms.
(b) Evaluate $f^{-1}(120)$, and say what it means in practical terms.
(c) Sketch a graph of f^{-1}.

10. When a cold yam is put into a hot oven to bake, the temperature of the yam rises. The rate, R (in degrees per minute), at which the temperature of the yam rises is governed by Newton's Law of Heating, which says that the rate is proportional to the temperature difference between the yam and the oven. The temparature of the yam will increase quickly at first, and then increases more and more slowly. If the oven is at 350^oF and the temperatue of the yam is H^oF.
(a) Write a formula giving R as a function of H.
(b) Sketch the graph of R against H.
(c) Sketch a graph of the temperature of the yam against time.
(d) Write a possible formula for H as a function of time t.

MAC 2311 Calculus 1 **Test 2** 25 Feb 1999

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

1. Find the area under the first arch (from 0 to $\sqrt{\pi}$) of $y = \sin(x^2)$ (State the numerical method you used.)

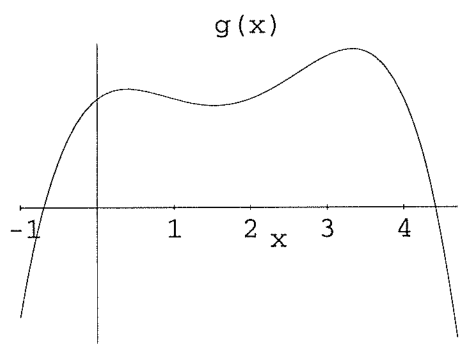
2. Find an equation for the tangent line to $f(x) = x^2$ at $x = 3$. Plot $f(x)$ and this tangent line.

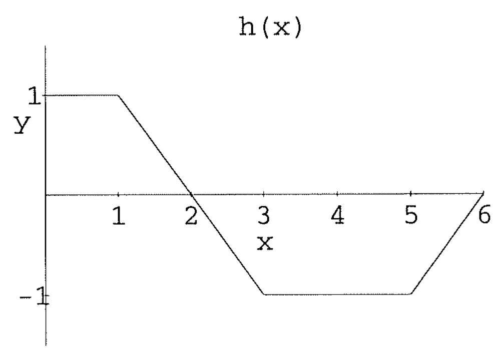
3. A car going 80 ft/sec (about 55 mph) brakes to a stop in 8 seconds. Its velocity is recorded every 2 seconds and is given in the following table
(a) Give your best estimate of the distance traveled by the car during the 8 seconds.
(b) To estimate the distanced traveled accurate to within 20 feet, how often should you record the velocity.

t(seconds)	0	2	4	6	8
$v(t)$(ft/sec)	80	52	28	10	0

4. For an even function f
(a) Suppose you know $\int_0^2 f(x)dx$. What is $\int_{-2}^2 f(x)dx$?
(b) Suppose you know $\int_0^5 f(x)dx$, and $\int_2^5 f(x)dx$. What is $\int_0^2 f(x)dx$?
(c) Suppose you know $\int_{-2}^5 f(x)dx$, and $\int_{-2}^2 f(x)dx$. What is $\int_0^5 f(x)dx$?

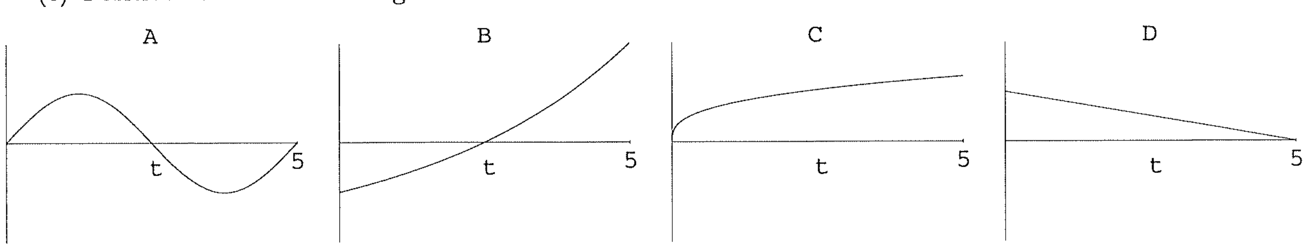
5. Sketch the graph of the derivative to the function $g(x)$ in the graph below. (You might want to trace $g(x)$ onto your answer sheet.)





6. The graph above plots the function $h(x)$. If $H' = h$ and $H(0) = 0$, find $H(b)$ for $b = 1, 2, 3, 4, 5, 6$.

7. Each of the graphs A-D below shows the position of a particle moving along the x-axis as a function of time, $0 \le t \le 5$. (The x-axis is vertical.) The vertical scales of the graphs are the same. During this time interval, which particle has
(a) Constant velocity?
(b) The greatest initial velocity?
(c) The greatest average velocity?
(d) Zero average velocity?
(e) Positive acceleration throughout?



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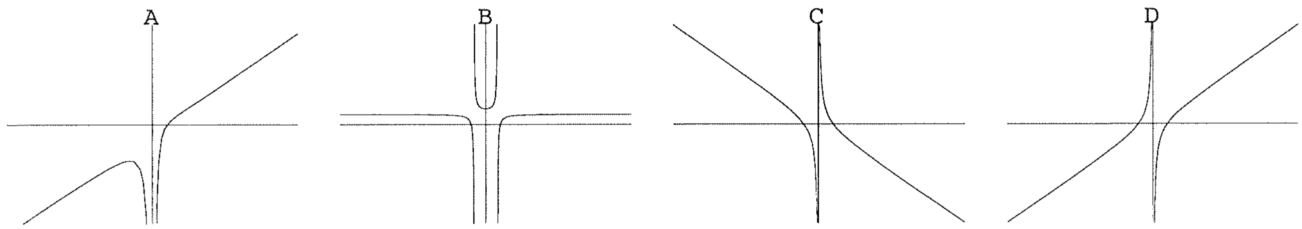
8. Match the following functions with the graphs below. Assume $0 < b < a$.

$$I.\quad y = \frac{a}{x} - x \qquad II.\quad y = \frac{(x-a)(x+a)}{x} \qquad III.\quad y = \frac{(x-a)(x^2+a)}{x^2} \qquad IV.\quad y = \frac{(x-a)(x+a)}{(x-b)(x+b)}$$



9. Values of three functions are given in the table below rounded to to two decimal places. One function is of the form $y = ab^t$, one is of the form $y = ct^2$ and one is of the form $y = kt^3$. Which function is which? And find the constants a, b, c and k.

t	$f(t)$	t	$g(t)$	t	$h(t)$
2.0	4.40	1.0	3.00	0.0	2.04
2.2	5.32	1.2	5.18	1.0	3.06
2.4	6.34	1.4	8.23	2.0	4.59
2.6	7.44	1.6	12.29	3.0	6.89
2.8	8.62	1.8	17.50	4.0	10.33
3.0	9.90	2.0	24.00	5.0	15.49

10. Define

$$f(x) = \frac{1}{1+e^{-x}}.$$

(a) Is f increasing or decreasing? Explain why f is invertiable.
(b) What is the domain of f^{-1}?
(c) Sketch the graphs of f and f^{-1} on the same axes, and explain their relationship.
(d) Find a formula for $f^{-1}(x)$.

MAC 2311 Calculus 1 **Test 3** 30 Mar 1999

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

1. Find $\frac{dy}{dt}$ if $y = (t^2 + 3t + 2)/(t + 1)$

2. If $g(s) = 2^s + \ln s + \arcsin s + \tan s - \pi^3$ find g'

3. Find w' if $w = 2\sin x + \sin(2x) + \sin(x^2) + \sin^2 x + \sin(\sin x)$

4. Find y' implicitly, if $x^3 + y^3 - 4x^2 y = 1$

5. Using L'Hopital, find

$$\lim_{x \to 0} \frac{\sin(2x)}{\tan(5x)}$$

6. Find the equation of the tangent line to $f(x) = \cos x$ at $x = \pi/3$ and find where it intersects the x-axis.

7. Suppose we are given the data in the table about the functions f and g and their derivatives. Find the following values:

(a) $h'(4)$ if $h(x) = f(g(x))$
(b) $h'(4)$ if $h(x) = g(f(x))$
(c) $h'(4)$ if $h(x) = f(x)g(x)$
(d) $h'(4)$ if $h(x) = f(x)/g(x)$
(e) $h'(4)$ if $h(x) = f(g(f(x)))$

x	1	2	3	4
$f(x)$	3	2	1	4
$f'(x)$	1	4	2	3
$g(x)$	2	1	4	3
$g'(x)$	4	2	3	1

8. Find the local linerization of $\sqrt{x}$ near $x = 1$. Use your approximation to estimate $\sqrt{.96}$. Find $\sqrt{.96}$ exactly, and determine the percent error of your estimate. [The percent error is the absolute value of the error divided by the exact value (with the answer written as a percentage).]

9. A spherical cell is growing at a constant rate of $400\mu m^3$/day ($1\mu m = 10^{-6} m$). At what rate is its radius increasing when the radius is $10\mu m$?

10. A radio navigation system used by aircraft give a cockpit readout of the distance, s, in miles, between a fixed ground station and the aircraft. The system also gives a readout of the instantaneous rate of change, ds/dt, of this distance in miles/hour. An aircraft on a straight flight path at a constant altitude of 10,560 feet (2 miles) has passed directly over the ground station and is now flying away from it. What is the speed of this aircraft along its constant altitude flight path when the cockpit readouts are $s = 4.6$ miles and $ds/dt = 210$ miles/hour. As part of your solution, draw a picture with at least labels for the aircraft, ground station and the distance s.