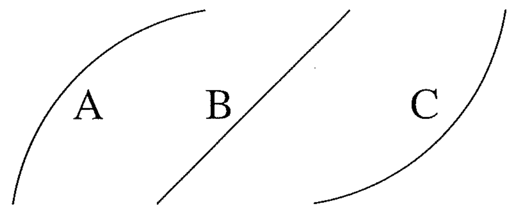
MAC 2311 Calculus 1 **Test 1** 21 Sep 1999

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Do **NOT** write on this page. Calculators can be used for graphing and calculating only. Give exact answers when possible.

1. Each of the functions in the table below is increasing, but in different ways. Which of the graphs below best fits each function.

t	$g(t)$	$h(t)$	$k(t)$
1	23	10	2.2
2	24	20	2.5
3	26	29	2.8
4	29	37	3.1
5	33	44	3.4
6	38	50	3.7

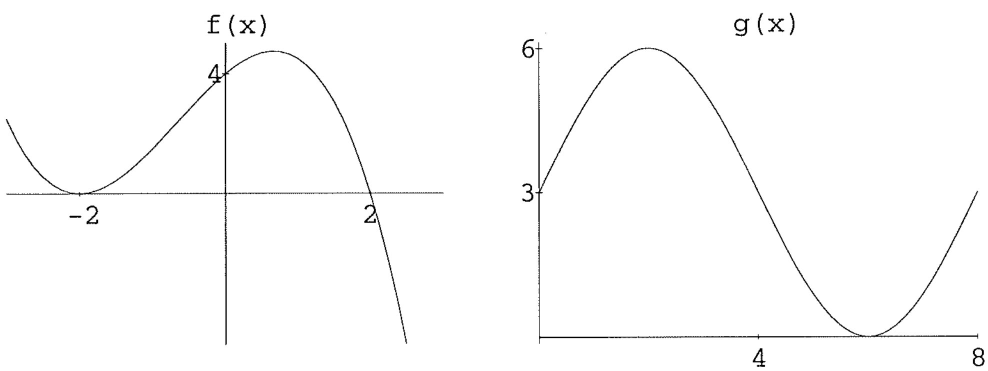


2. Suppose $C = f(A)$, where C is the cost in dollars, of building a store of area A square feet. Explain the meaning of $f(10000) = 350000$ and $f^{-1}(20000) = 4000$ in practical terms.

3. In each pair below, which function will eventually be larger as x goes to infinity?
(a) $2x^5$ or $200x^4$ (b) $10x^3$ or e^x (c) x^{-2} or x^{-5} (d) $x^{1/2}$ or $\ln x$ (e) x^π or $(1.0000001)^x$

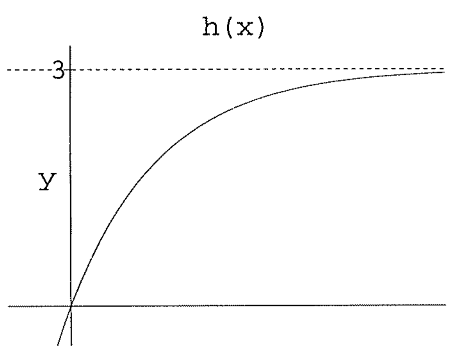
4. After 100 years radioactive substance is reduced to 1/5 the initial amount, how long until only 1/20 of the initial amount remains?

5. Determine the cubic polynominal that represents the graph of $f(x)$ below.



6. Find a possible formula for the graph of $g(x)$ above. Give its amplitude and period.

7. Find a possible equation involving an exponential for the graph of $h(x)$ below. Note that (0,0) is on the curve.



Welcome to test two side two.

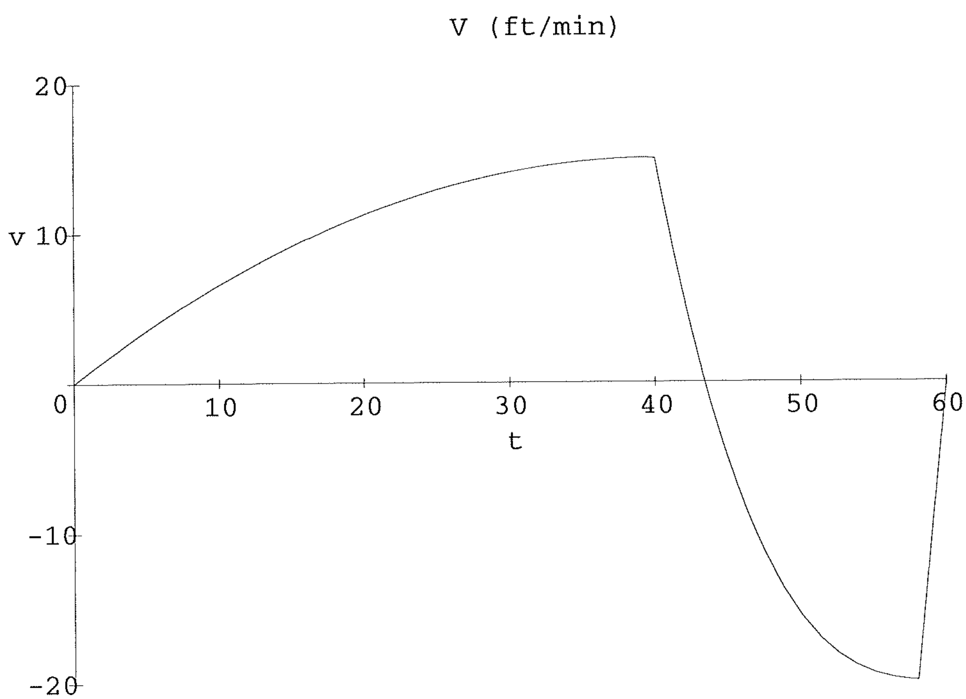
8. Find the derivative of $f(x) = 1/x$ algebraically.

9. Students were asked to evaluate $f'(4)$ from the following table.

x	1	2	3	4	5	6
$f(x)$	4.2	4.1	4.2	4.5	5.0	5.7

- Student A estimated the derivative as $f'(4) \approx \frac{f(5)-f(4)}{5-4} = 0.5$.
- Student B estimated the derivative as $f'(4) \approx \frac{f(4)-f(3)}{4-3} = 0.3$.
- Student C suggested that they should split the difference and estimate the average of these two results, that is, $f'(4) \approx \frac{1}{2}(0.5 + 0.3) = 0.4$.

(a) Sketch the graph of $f(x)$ and indicate how these three estimates are represented on the graph.
(b) Explain which answer is likely to be best.
(c) Use Students C's method to find an algebraic formula which approximates $f'(x)$ using increments of size h.



10. The Montgolfier brothers (Joseph and Etienne) were eighteenth-century pioneers in the field of hot-air balloning. Had they had the appropriate instruments, they might have left us a record of one of their early experiments, like the one show above. The graph show their vertical velocity, v, with upward as positive.
(a) Over what intervals is the acceleration positive? Negative?
(b) What was the greatest altitude achieved, and at what time?
(c) At what time was the deceleration the greatest?
(d) What might have happend during the this flight to explain the answer to part (c)?
(e) This particular flight ended on top of a hill. How do you know that it did and what was the height of the hill above the starting point?

MAC 2311 Calculus 1 **Test 2** 19 Oct 1999

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1. Find $\int_1^2 x^x\,dx$ using your calculator. (State the numerical method you used and the model of your calculator.)

2. Find an equation for the tangent line to $f(x) = 1/x$ at $x = 2$. Plot $f(x)$ and this tangent line.

3. You jump out of an airplane. Before your parachute opens you fall faster and faster, but your acceleration decreases as you fall because of air resistance. The table below gives your acceleration, a (in m/sec^2), after t seconds.

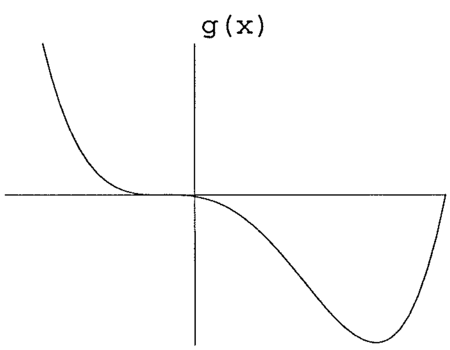
t(seconds)	0	1	2	3	4	5
$a(t)$(ft/sec^2)	9.81	8.03	6.53	5.38	4.41	3.61

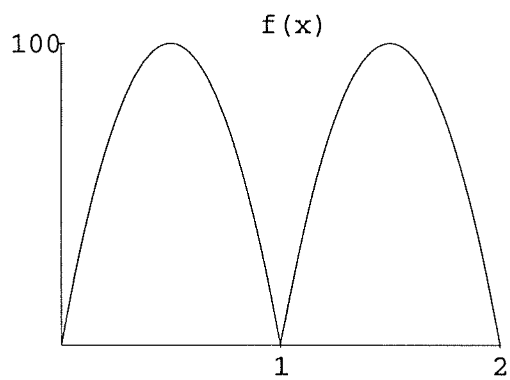
(a) Give upper and lower estimates of your speed at $t = 5$.

(b) Get a new estimate by taking the average of your upper and lower estimates. What does the concavity of the graph of acceleration tell you about your new estimate?

4. The temperature, T, in degrees Fahrenheit, of a cold yam places in a hot oven is given by $T = f(t)$, where t is the time in minutes since the yam was put in the oven. What is the sign of $f'(t)$ and why? What are the units of $f'(20)$? What is the practical meaning of the statement $f'(20) = 2$?

5. Sketch the graph of the derivative to the function $g(x)$ in the graph below. (You might want to trace $g(x)$ onto your answer sheet.)





6. The graph of the continuous function $f(x)$ is given above. Rank the following integrals in ascending numerical order. Explain your reasons.

(i) $\int_0^2 f(x)dx$ (ii) $\int_0^1 f(x)dx$ (iii) $\int_0^2 (f(x))^{1/2}dx$ (iv) $\int_0^2 (f(x))^2 dx$.

7. There is a function called the error function, $y = erf(x)$. Suppose your calculator has a button for $erf(x)$ that gives the following values:

$erf(0) = 0$ $\quad erf(1) = 0.84270079$ $\quad erf(0.1) = 0.11246292$ $\quad erf(0.01) = 0.01128342$

(a) Use all this information to determine your best estimate for $erf'(0)$. (Give only those digits of which you feel reasonably certain.)

(a) Suppose you find that $erf(0.001) = 0.00112838$. How does this extra information change your answer to part (a)?

There is more test on the other side.

Welcome to test two side two.

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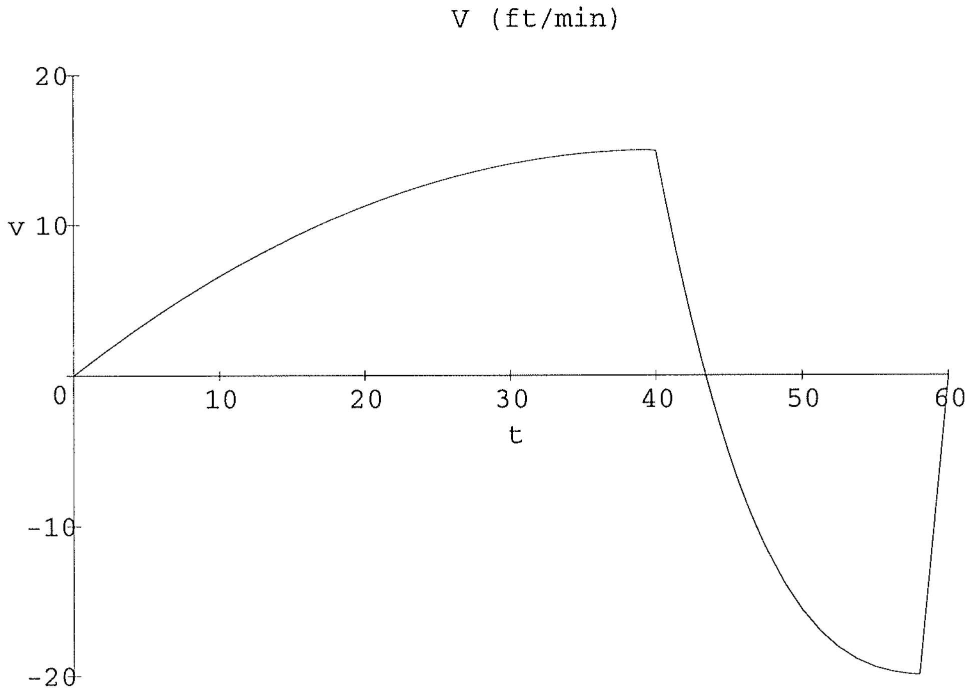

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(e) This particular flight ended on top of a hill. How do you know that it did and what was the height of the hill above the starting point?

MAC 2311 Calculus 1 **Test 3** 2 Dec 1999

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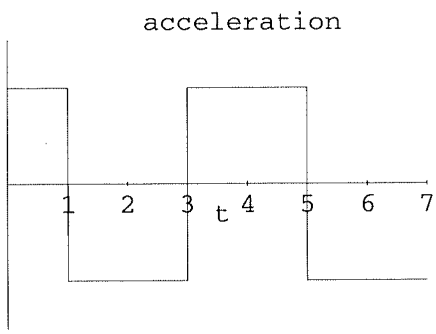
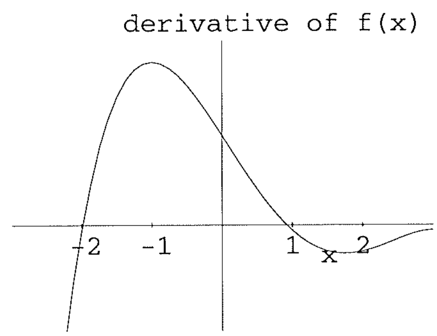
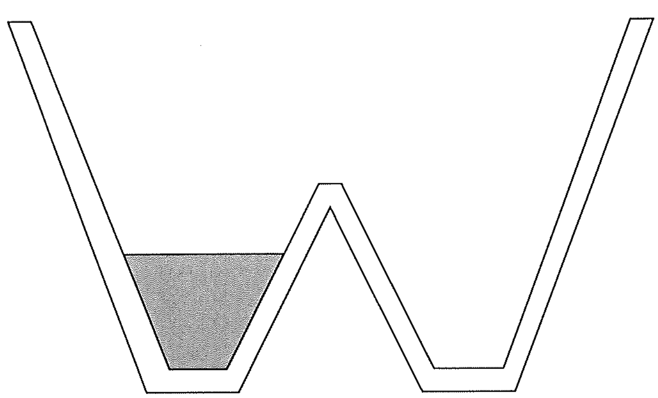
1. Let $f(x) = \sin(x) + 2^x + \sinh(x) + \sqrt{x} + \pi$
   a. Find $f'(x)$.
   b. Find $\int f(x)dx$.
2. Given $r(2) = 4, s(2) = 1, s(4) = 2, r'(2) = -1, s'(2) = 3, s'(4) = 3$. Compute the following derivatives, or state what additional information you would need to be able to compute the derivative.
   a. $H'(2)$ if $H(x) = r(x) + 2s(x)$
   b. $H'(2)$ if $H(x) = r(x) \cdot s(x)$
   c. $H'(2)$ if $H(x) = \sqrt{r(x)}$
   d. $H'(2)$ if $H(x) = r(s(x))$
   e. $H'(2)$ if $H(x) = s(r(x))$

3. The acceleration, a, of a particle as a function of time t is shown in the graph below (left). Sketch graphs of the velocity and position against time. Assume the particle starts at rest at the origin.





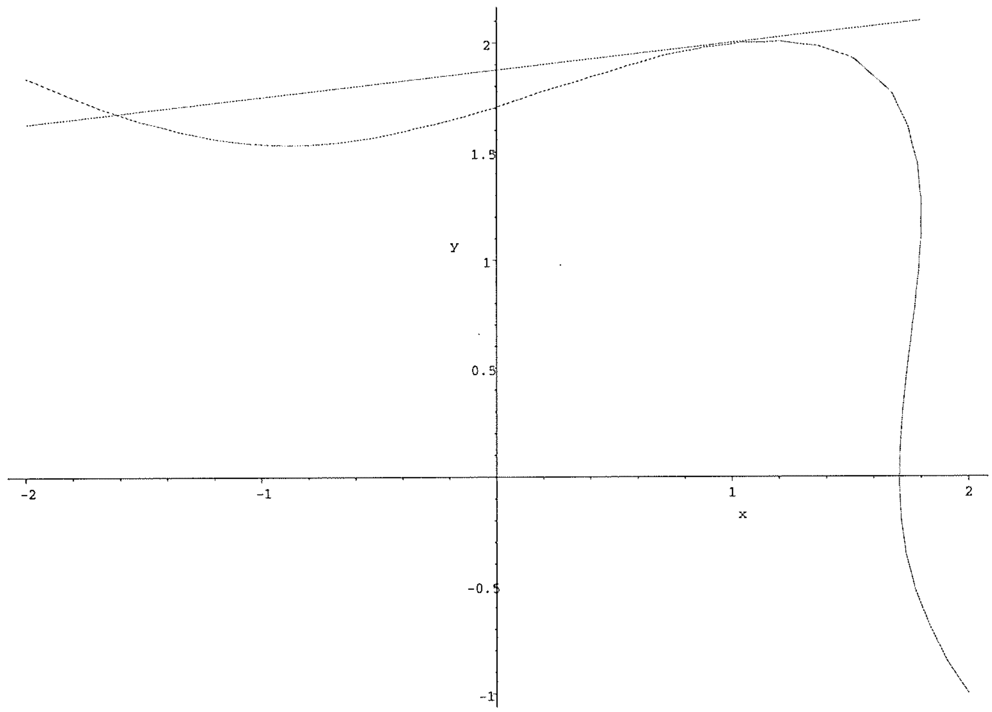
4. The graph above (right) plots the derivative of the function $f(x)$. Sketch a possible graph for $f(x)$. Mark the points, $x = -2, -1, 1, 2$ on your graph and label local maxima, local minima and points of inflection.



5. Water flows at a constant rate into the left side of the W-shaped container shown above. Sketch a graph of the the height H, of the water in the left side of the container as a function of time, t. Suppose the container starts out empty. [Make sure your function has the proper concavity. Yes, the water does start filling the right side when the height is high enough.]

There is more test on the other side.

Welcome to test three side two.

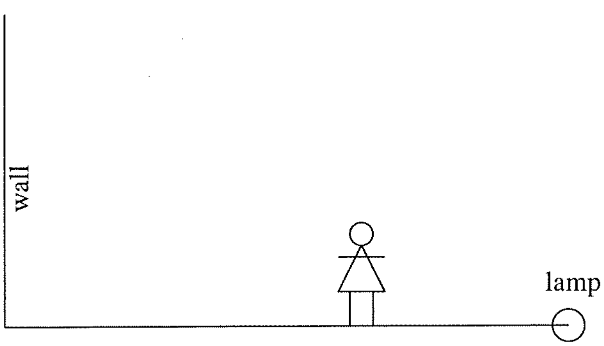


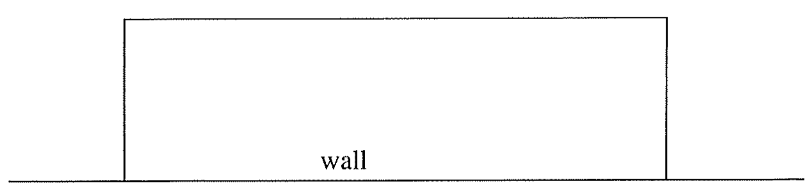
6. Consider the equation $x^3 + y^3 - xy^2 = 5$ (see graph above).
   a. Find dy/dx by implicit differentiation.
   b. Find the equation of the tangent line to the curve when $x = 1$ and $y = 2$, (see graph above).

7. Find $\lim_{x \to 0^+} x \ln x$ using L'Hopital rule. [Hint: $x \ln x = \frac{\ln x}{1/x}$.]

8. Find the (global) minimum and maximum values of the function $f(x) = x + \sin x$ for $0 \le x \le 2\pi$.

9. A floor lamp is 30 feet away from a high wall and directly between the lamp and the wall is a 6 foot women 20 feet from the wall (see below left). How fast is her shadow changing if the lamp is moving 2 feet per second away from the wall?





10. If you have 100 feet of fencing and want to enclose a rectanglar area up against a long, straight wall, (see above right,) what is the largest area you can enclose?