REAL ANALYSIS MAT 530A-B Do as many Problems as time allows.

1. If $a, b$ are real numbers and if for each $\varepsilon > 0$, $a \le b + \varepsilon$, then show $a \le b$.

2. Show $\dfrac{h}{1+h^2} < \arctan h < h$, for $h > 0$.

3. Let $f_n : \mathbb{R} \to \mathbb{R}$, $n = 1, 2, \ldots$ be a sequence of continuous function and suppose $g : \mathbb{R} \to \mathbb{R}$ is the <u>uniform</u> limit of $\{f_n\}$. Show that $g$ is a continuous function.

4. Suppose $f(x) \ge 0$, $f \not\equiv 0$, and $f(x)$ is continuous. Show that there are $a$, $\delta > 0$, $\varepsilon > 0$ such that

$$|x - a| < \delta \implies f(x) \ge \varepsilon.$$

5. Show $f(x)$ has a power series expansion for (at least) $|x| < 1$, if and only if $f(x)/1-x$ has a power series expansion for $|x| < 1$.

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For 6, 7, 8:

$$\text{Let } F(x) = \begin{cases} 0 & x \text{ is irrational} \\ \frac{1}{q} & x \text{ is rational and } x = p/q \text{ in lowest terms } (p \,\&\, q \text{ integers} \end{cases}$$

6. Show $F(x)$ is continuous only on the irrationals

7. Show $F(x)$ is Riemann integrable on $[0,1]$ and $\int_0^1 F(t)\,dt = 0$

8. Show $F(x)$ is not differentiable anywhere [HINT: #6 takes care of the rations and the following picture may help for the irrationals ]