MATH 431 – 530A True (T) or False (F) −2: for each blank

−4: for each incorrect answer score range −100 to 100.

Make 5 vertical columns of 10 each: 1–10, 11–20, 21–30, 31–40, 41–50

1. $0=1$.
2. A convergent sequence is a Cauchy sequence.
3. Every continuous function on a closed interval has a maximum value.
4. If $a_n > 0$ for all n, then $\lim\limits_{n\to\infty} a_n = +\infty$ if and only if $\lim\limits_{n\to\infty} 1/a_n = 0$.
5. For $0<a<b$, $\sqrt{a+b} \le \sqrt{a} + \sqrt{b}$.
6. The limsup of the sequence $2, -1, 1\frac{1}{2}, -\frac{1}{2}, 1\frac{1}{4}, -\frac{1}{4}, \ldots$ is 2.
7. A unbounded function can be integrable.
8. The rationals are dense in the reals.
9. $f(x)$ is not uniformly continuous on $(-\infty, +\infty)$ if $\forall \varepsilon>0\ \exists \delta>0\ \exists x_1, x_2$ with $|x_1 - x_2| < \delta$ but $|f(x_1) - f(x_2)| \ge \varepsilon$.
10. $\limsup\limits_{n\to\infty} \{a_n\} \le \liminf\limits_{n\to\infty} \{a_n\}$.
11. If A is a finite set of reals and $\inf\{|a| : a\in A\} = 0$ then $0 \in A$.
12. If $\lim\limits_{n\to\infty} |a_n| = a$, then $\lim\limits_{n\to\infty} a_n = a$ or $-a$.
13. If $0<a<b$ $\sqrt{ab} \le \frac{1}{2}(a+b)$.
14. A strictly increasing function always has a continuous inverse.
15. $f(x) = \dfrac{x^2-9}{x+3}$ is not continuous at $x=-3$.
16. To Show $A \Rightarrow B$ or C, it is enough to show $(A \text{ and not } B) \Rightarrow C$.
17. Suppose $f(x)$, $f'(x)$ are both continuous on $[a,b]$, then for each $\xi \in (a,b)$, there are c, d with $a<c<d<b$ and
$$f'(\xi) = \frac{f(d)-f(c)}{d-c}$$
18. If $\lim\limits_{x\to a} \varphi(x) = L$, $\lim\limits_{x\to L} f(x) = K$, then $\lim\limits_{x\to a} f(\varphi(x)) = K$.
19. $f(x) = x\sin(1/x)$ can be defined at $x=0$ so to be continuous everywhere.
20. For $0<a<b$ $\dfrac{1}{2\sqrt{a}} < \dfrac{\sqrt{b}-\sqrt{a}}{b-a} < \dfrac{1}{2\sqrt{b}}$.

21. If $|f'(x)| \le M$ for $x \in [a,b]$, then $f(x)$ is uniformly continuous on $(a,b]$.

22. $f(x)$ is continuous at $x=a$ if and only if for all $a_n \to a$, $f(a_n) \to f(a)$.

23. If $f(x)$ is continuous at $x=a$, $g(x)$ is continuous at $x=b$ and $a=g(b)$, then $f(g(x))$ is continuous at $x=b$.

24. The set $\{ k/2^n : k = 0, \pm 1, \pm 2, \ldots ; n = 0, 1, 2, \ldots \}$ is not dense in the reals.

F 25. If $a_n \le b_n \le c_n$, and $\lim_{n\to\infty} (c_n - a_n) = 0$, then $\{b_n\}$ converges.

26. ~~For~~ $||a| - |b|| \le |a-b|$.

27. If $a_n \to 0$ and $b_n \to +\infty$, then $a_n b_n \to 1$.

28. If $A \Rightarrow B$, then $(\text{not } A) \Rightarrow (\text{not } B)$.

T 29. If $\varphi(x)$ is a step function and $f(x) = \int_0^x \varphi(t)\,dt$, then $f(x)$ is a continuous function.

30. If B is a set of reals, and a is a real number, and $\forall \varepsilon > 0$ $\exists b \in B$ with $b < a + \varepsilon$, then $\exists b \in B$ with $b \le a$.

31. If $\lim_{n\to\infty} (a_n + b_n)$ exists, then so does $\lim_{n\to\infty} a_n$ or $\lim_{n\to\infty} b_n$ also exists.

32. Every subsequence of a ~~convergente~~ convergent sequence converges to the same limit.

T 33. The function $g(x)$ is in $\mathcal{I}[a,b]$ if for all $\varepsilon > 0$ there are $f(x)$, $h(x)$ both in $\mathcal{I}[a,b]$ with $f(x) \le g(x) \le h(x)$, and $\int_a^b (g(x) - f(x))\,dx < \varepsilon$.

34. Suppose $F(x) = f(x) - g(x)$, $F(0) = F'(0) = 0$ and $F''(x) > 0$ for $x > 0$, then $f(x) > g(x)$ for $x > 0$.

35. Some Cauchy sequences do not converge.

F 36. $f(x) = \begin{cases} 1 & x \text{ irrational} \\ 0 & x \text{ rational} \end{cases}$ is continuous at $x = \sqrt{2}$.

37. $f(x)$ in #36 is in $\mathcal{I}[0,1]$.

38. Bounded sequences converge.

39. If $f(x)>0$ and $f'(x)<0$ for $x>0$ then $\lim_{x\to+\infty} f(x)$ exists.

40. For any sequence $\{a_n\}$ either

(1) $\lim_{n\to\infty} a_n = A$ where $A \in \mathbb{R}$ or $A = \pm\infty$.

or (2) there are subsequences $\{a_{n_i}\}$ and $\{a_{n_j}\}$ with $\lim_{i\to\infty} a_{n_i} = B$ and $\lim_{j\to\infty} a_{n_j} = C$, $B \neq C$ where B, C are in $\mathbb{R}$ or $= \pm\infty$.

41. If $\delta = \delta(\varepsilon) = \sqrt{\varepsilon}$, and $f(x) = x^2$, then at $a=2$ ~~x=2~~ $|x-a|<\delta \Longrightarrow |f(x)-f(a)|<\varepsilon$.

42. $f(x) = \sin\frac{1}{x}$ is in $\mathcal{R}[0,1]$.

43. If $\varphi(x) \geq 0$, $\varphi(x)$ a step function, but $\varphi(x) \not\equiv 0$ then $\int_a^b \varphi(x)\,dx > 0$ $(a<b)$

44. If $0 \leq a \leq b$ $\lim_{p\to+\infty} (a^p+b^p)^{\frac{1}{p}} = \max(a,b)$.

45. If $p(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0 > 0$ for all x and $a_n \neq 0$, then n must be even.

46. A sequence can have only a finite number of limit points.

47. If $f(x)$ is a function and $f'(x)$ is defined and $= 0$ everywhere but at $x = a_1$, $x = a_2$, ... and $x = a_n$, then $f(x)$ is a constant, except maybe at $x = a_1$, $x = a_2$, ... and $x = a_n$.

48. If $a_n \leq b_n$, then $\lim_{n\to\infty} a_n \leq \lim_{n\to\infty} b_n$, if both limits exist.

49. If A, B are sets of real numbers and $A \supset B$ then $\sup A \geq \sup B$.

50 $|a+b| \leq |a|+|b|$.