1 Thm: For [illegible]

$$(*) \quad \left(\sum_{i=1}^{n} (a_i+b_i)^2\right)^{\frac{1}{2}} \le \left(\sum_{i=1}^{n} a_i^2\right)^{\frac{1}{2}} + \left(\sum_{i=1}^{n} b_i^2\right)^{\frac{1}{2}}.$$

<u>Proof(?)</u>

I Since all the roots are positive and of positive numbers $(*)$ is equivalent of its square:

$$\sum_{i=1}^{n} (a_i+b_i)^2 \le \sum_{i=1}^{n} a_i^2 + \sum_{i=1}^{n} b_i^2 + 2\left(\sum_{i=1}^{n} a_i^2 \cdot \sum_{i=1}^{n} b_i^2\right)^{\frac{1}{2}}$$

II $$\sum_{i=1}^{n} (a_i+b_i)^2 = \sum_{i=1}^{n} a_i^2 + \sum_{i=1}^{n} b_i^2 + 2\sum_{i=1}^{n} a_i b_i$$

III hence I is equivalent to

$$2\sum_{i=1}^{n} a_i b_i \le 2\left(\sum_{i=1}^{n} a_i^2\right)^{\frac{1}{2}}\left(\sum_{i=1}^{n} b_i^2\right)^{\frac{1}{2}}$$

IV Now on the homework we have shown

$$\left(\sum_{i=1}^{n} a_i b_i\right)^2 \le \left(\sum_{i=1}^{n} a_i^2\right)\left(\sum_{i=1}^{n} b_i^2\right)$$

which clearly implies III

V The proof is done.