MATH 431 – 530A TAKE HOME FINAL DUE: 19 MAR 1976 @ HIGH NOON

1. Suppose $\lim_{n\to\infty} a_n = A$, let $s_n = \frac{1}{n}\sum_{k=1}^{n} a_k$ show $\lim_{n\to\infty} s_n = A$. Give a counterexample to the converse.

2. Suppose $p > 1$, K a constant and $f(x)$ a function which satisfies $|f(s) - f(t)| \le K|s-t|^p$ for each s & t in $[a,b]$. Show $f(x)$ is constant on $[a,b]$. HINT: Compute $f'(x)$.

3. Suppose $\lim_{n\to\infty} b_n = 0$ and $\sum_{k=1}^{\infty} a_k$ converges, show $\sum_{k=1}^{\infty} a_k b_k$ converges.

4. If $a_k \ge 0$ and $\sum_{k=1}^{\infty} a_k$ converges, then $\sum_{k=1}^{\infty} (a_k)^2$ converges. Show by example $\sum (a_k)^2$ can converge does not imply $\sum a_k$ converges (for $a_k \ge 0$)

5. If f is a continuous function on $[0,1]$ show
$$\int_0^1 |f(x)|\,dx \le \left\{\int_0^1 |f(x)|^2 dx\right\}^{\frac{1}{2}}.$$

6. Show that for all x: $e^x + e^{-x} \le 2e^{x^2}$.

7. If $f(x)$ is the derivative of some function $g(x)$, on an interval, then $f(x)$ assumes (as a value) every number between any two of its values. CAUTION: $f(x)$ need not be continuous.

8. Define $\prod_1^\infty a_k = \lim_{n\to\infty} \prod_1^n a_k$ (if it exists) where $\prod_1^n a_k = a_1 a_2 a_3 \cdots a_{n-1} a_n$ (the product of the 1st n-terms) Show if $a_k \ge 0$ and $\sum_{k=1}^{\infty} a_k$ converges then $\prod_1^\infty (1 + a_k)$ converges. HINT: log both sides & show $\ln(1+x) \le x$ $x \ge 0$.