MAP 2302 ODE Homework Summer 2020

Based on Boyce and DiPrima 8th Edition

June 16, 2020

1 Homework 01

Section 2.1 Integrating Factors 1c, 4c, 7c, 14, 26b, 30

2.1.1c Find the general solution to the given equation and use it to determine how solutions behave as $t \to \infty$

$$y' + 3y = t + e^{-2t}$$

2.1.4c Find the general solution to the given equation and use it to determine how solutions behave as $t \to \infty$

$$y' + (1/t)y = 3\cos 2t$$

2.1.7c Find the general solution to the given equation and use it to determine how solutions behave as $t \to \infty$

$$y' + 2ty = 2te^{-t^2}$$

2.1.14 Find the solution to the given initial value problem

$$y' + 2y = 2te^{-2t} \qquad y(1) = 0$$

2.1.26b Find the solution to the given initial value problem and find a_0 where the solutions as $t \to 0$ change behavior.

$$(\sin t)y' + (\cos t)y = e^t \qquad y(1) = a, \qquad 0 < t < \pi$$

2.1.30 Find the value of y_o for which the solution to the initial value problem below remains finite as $t \to \infty$

$$y' - y = 1 + 3\sin t \qquad y(0) = y_0$$

2 Homework 02

Section 2.2 Separable Equations 1, 6, 9ac, 23

2.2.1 Solve $y' = x^2/y$

2.2.6 Solve $xy' = (1 - y^2)^{1/2}$

2.2.9ac Find the solution of the initial value problem

$$y' = (1-2x)y^2, \qquad y(0) = -1/6$$

and determine the interval the solution is defined.

2.2.23 Solve the initial value problem

$$y' = 2y^2 + xy^2, \qquad y(0) = 1$$

and determine where the solution attains its minimum value.

3 Homework 03

Section 2.3 Modeling 1, 5, 16 and vocab.pdf

2.3.1 Consider a tank used in certain hydrodynamic experiments. After one experiment the tank contains 200 liters of a dye solution with a concentration of 1 g/liter. To prepare for the next experiment, the tank is to be rinsed with fresh water flowing in at a rate of 2 liter/min, the well-stirred solution flowing out at the same rate. Find the time that will elapse before the concentration of dye in the tank reaches 1% of its original value.

2.3.5 A tank contain 100 gallons of water and 50 oz of salt. Water containin a salt concentration of $\frac{1}{4}(1 + \frac{1}{2}\sin t)$ oz/gal flows into the tank at a rate of 2 gal/min, and the mixture in the tank flows out at the same rate.

a. Find the amount of salt in the tank at any time.

b. Plot the solution for a time period long enough so that you can see the ultimate behavior of the graph.

c. The long-time behavior of the solution is an oscillation about a certain constant level. What is this level? What is the amplitude of the oscillation?

2.3.16 Newton's law of cooling states that the temperature of an object changes at a rate proportional to the difference between its temperature and that of its surroundings. Suppose that the temperature of a cup of coffee obeys Newton's law of cooling. If the coffee has a temperature for 200 deg F when freshly poured, and 1 min later has cooled to 190 deg F in a room at 70 degF, determine when the coffee reaches a temperature of 150 deg F.

4 Homework 04

Section 2.3 Modeling 6, 19 and Section 2.4 Linear vs Non-linear 1, 7, 13, 22

2.3.6 Suppose that a tank containing a certain liquid has an outlet near the bottom. Let $h(t)$ be the height of the liquid surface above the outlet at time t. Torricelli's principle states that the outflow velocity v at the outlet is equal to the velocity of a particle falling freely (with no drag) from the height h.

a. Show that $v = \sqrt{2gh}$, where g is the acceleration due to gravity.

b. By equation the rate of outflow to the rate of change of the liquid in the tank, show that $h(t)$ satisfies the equation

$$A(h)\frac{dh}{dt} = -\alpha a\sqrt{2gh},$$

Where $A(h)$ is the area fo the cross section of the tank at height h and a is the area of the outlet. The constant α is a contraction coefficient that accounts for the observed fact that the cross of the (smooth) outflow stream is smaller that a. The value of α for water is about 0.6.

c. Consider the water tank in the form of a right cicular cylinder that is 3 m high above the outlet. The radius of the tank is 1 m and the radius of the outlet is 0.1 m. If the tank is initially full of water, determine how long it takes to drain the tank down to the level of the outlet.

2.4.1 Determine without solving an interval in which the solution of the initial value problem is certain to exist.

$$(t-3)y' + (\ln t)y = 2t, \qquad y(1) = 2.$$

2.4.7 State where the ty-plane the hypothesis of Theorem 2.4 are satisfied.

$$y' = \frac{t-y}{2t+5y}$$

2.4.13 Solve the given initial value problem and determine how the interval in which the solution exists depends on the initial value y_0.

$$y' = -4t/y, \qquad y(0) = y_0$$

2.4.22 a. Varify that both $y_1(t) = 1-t$ and $y_2(t) = -t^2/4$ are solutions to the initial value problem:

$$y' = \frac{-t+(t^2+4y)^{1/2}}{2}, \qquad y(2) = -1.$$

Where are these solutions valid?

b. Explain why the existence of two solutions of the given problem does not contradict the uniqueness part of 2.4.2.

c. Show that $y = ct + c^2$, where c is an arbitrary constant satisfies the differential equation in part a for $t \geq -2c$. If $c = -1$, the initial condition is also satisfied, and the solution $y = y_1(t)$ is obtained. Show there is no choice of c that gives the section solution $y = y_2(t)$.

5 Homework 05

Section 2.3 Modeling 2, 4 #10 from this old test #10 from this other old test

2.3.2 A tank initially contains 120 liters of pure water. A mixture containing a concentration of γ g/liter of salt enters the tank at a rate of 2 liters/min, and the well-stirred mixture leaves the tank as the same rate. Find an expression in terms of γ for the amount of salt in the tank at any time t. Also find the limiting amount of salt in the tank as $t \to \infty$

2.3,4 A tank with a capacity of 500 gal originall contains 200 gal of water with 100 lb of salt in solution. Water containing 1 lb of salt per gallon is entering at a rate of 3 gal/min, and the mixtuer is allowed to flow out of the tank at a rate of 2 gal/min. Find the amount of salt in the tank at any time prior to the instant when the solution begins to overflow. Find the concentration (in pounds per gallon) of salt in the tank when it is on the point of overflowing. Compare this concentration with the theoretical limiting concentration if the tank had infinite capacity.

6 Homework 06

Section 2.5 Autonomous equations 10, 14, 16b, 18ab

2.5.10 $f(y) = y(1-y^2), \qquad -\infty < y < \infty$. Sketch the graph of $f(y)$ versus y, determine the critical (equilibrium) points and classify each one as asymptotically stable, unstable, or semistable. Draw the phase line and sketch several graphs of solution to

$$\frac{dy}{dt} = f(y)$$

in the ty-plane.

2.5.14 Consider the equation $dy/dt = f(y)$ and suppose y_1 is a critcal point, that is $f(y_1) = 0$. Show that the constant solution $\phi(t) = y_1$ is asymptotically stable if $f'(y_1) < 0$ and unstable if $f'(y) > 0$.

2.5.16b Another equation that has been used to model population growth is Gompertz equation:

$$dy/dt = ry\ln(K/y),$$

where r and K are positive constants. For $0 \le y \le K$, determine where the graph of y versus t is concave up and where it is concave down.

2.5.18ab A pond forms as water collects in a conical depression of radius a and depth h. Suppose that water flows in at a constant rate k and is lost through evaporation at a rate proportional to the surface area.

a. Show that the volume $V(t)$ of the water in the pond at time t satisfies the differential equation

$$dV/dt = k - \alpha\pi(3a/\pi h)^{2/3}V^{2/3}$$

where α is the cofficient of evaporation.

b. Find the equilibrium depth of the water in the pond. Is the equilibrium asymptotically stable?

7 Homework 07

Section 2.6 Exact only 2, 10, 13

2.6.2 Is the equation below exact? If so find the solution.

$$(2x + 4y) + (2x - 2y)y' = 0.$$

2.6.10 Is the equation below exact? If so find the solution.

$$(y/x + 6x)dx + (\ln x - 2)dy = 0.$$

2.6.13 Solve the inital value problem and determine at least approximately where the solution is valid.

$$(2x - y)dx + (2y - x)dy = 0.$$

8 Homework 08

Section 2.7 do Euler part of this url Euler Method Problems

9 Homewort 09

Review? 2.1 17, 19; 2.2 14, 16, 2.6 14, 16

2.1.17 Find the solution of the initial value problem

$$y' - 2y = e^{2t}, \qquad y(0) = 2.$$

2.1.19 Find the solution of the initial value problem

$$t^3y' + 4t^2y = e^{-t}, \qquad y(-1) = 0, \quad t < 0$$

2.2.14 Solve

$$y' = xy^3(1+x^2)^{-1/2} \qquad y(0) = 1$$

and plot and find the interval in which the solution is defined.

2.2.16 Solve

$$y' = x(x^2+1)/4y^3 \qquad y(0) = 1$$

and plot and find the interval in which the solution is defined.

2.6.14 Solve and determine where the solution is valid

$$(9x^2 + y - 1)dx - (4y - x)dy = 0$$

2.6.16 Solve and determine where the solution is valid

$$(ye^{2xy} + x)dx + bxe^{2xy}dy = 0$$

10 Homework 10

Section 3.1 Homogeneous Equations with Constand Coefficients 1, 16, 22, 13, 28 and lab 1?

3.1.1 Find the general solution to

$$y'' + 2y' - 3y = 0$$

3.1.13 Find the general solution to given initial value problem. Sketch the graph of the solution and describe its behavior as t increases

$$y'' + 5y' + y = 0 \qquad y(0) = 1, \quad y'(0) = 0$$

3.1.16 Find the general solution to given initial value problem. Sketch the graph of the solution and describe its behavior as t increases

$$4y'' - y = 0 \qquad y(-2) = 1, \quad y'(-2) = -1$$

3.1.22 Solve the initial value problem

$$4y'' - y = 0 \qquad y(0) = 2, \quad y'(0) = \beta$$

. Then find β so that the solution approaches zero as $t \to \infty$

3.1.28 Consider the equation $ay'' + by' + cy = 0$, where $a, b,$ and c are constants with $a > 0$. Find conditions on $a, b,$ and c such that the roots of the characteristic equation are:

a. real, different and negative.

b. real with opposite signs.

c. real, different and positive.

11 Homework 11

Section 3.2 Fundamental Solutions of Linear Homogeneous Equations 3, 9, 21, 25

3.2.3 Find the Wronskian of the pair of fuctions $e^{-2t}, \quad te^{-2t}$

3.2.9 Determine the longest interval which the inital problem

$$t(t-4)y'' + 3ty' + 4y = 2, \qquad y(3) = 0, \quad y'(3) = -1$$

is certain to have a solution. Do not attempt to find the solution.

3.2.21 Find the fundamental set of solutions (Thm 3.2.5) for the point $t_0 = 0$ and the differential equation

$$y'' + y' - 2y = 0$$

3.2.25 Verify that $y_1(x) = x$ and $y_2(x) = xe^x$ are solutions to the differential equation

$$x^2y'' - x(x+2)y' + (x+2)y = 0, \; x > 0$$

Do they constitue a fundamental set of solutions?

12 Homework 12

Section 3.3 Linear Independence and the Wronskian 1, 2, 9, 15, 20, 27 Section 3.5 Repeated roots 1, 11, 14

3.3.1 Are $f(t) = t^2 + 5t$ and $g(t) = t^2 - 5t$ linearly independent or linearly dependent.

3.3.2 Are $f(\theta) = \cos 2\theta - 2\cos^2\theta$ and $g(\theta) = \cos 2\theta + 2\sin^2\theta$ linearly independent or linearly dependent.

3.3.9 The Wronskian of two functions is $W(t) = t\sin^2 t$. Are the functions linearly independent or linearly dependent? Why?

3.3.15 Find the Wronskian of two solution to equation without solving it:

$$t^2y'' - t(t+2)y' + (t+2)y = 0.$$

3.3.20 If y_1 and y_2 are linearly independent solutions of $ty'' + 2y' + te^ty = 0$ and $W(y_1, y_2)(2) = 3$, find the value of $W(y_1, y_2)(4)$.

3.3.27 Show t and t^2 are linearly independent on $-1 < t < 1$; indeed, they are linearly independent on every interval. Show also the $W(t, t^2) = 0$ at $t = 0$. What can you conclude from this about the possibility that t and t^2 are solutions of the equation

$$y'' + p(t)y' + q(t) = 0?$$

Verify that t and t^2 are solutions to the equation

$$t^2y'' - 2ty' + 2y = 0.$$

Doed this contradict your conclusions? Does the behavior of the Wronskian of t and t^2 contradict Thm 3.3.2?

3.5.1 Find the general solution to

$$y'' - 2y' + y = 0$$

3.5.11 Solve the inital value problem

$$9y'' - 12y' + 4y = 0 \qquad y(0) = 2, \quad y'(0) = -1.$$

3.5.14 Solve the inital value problem

$$y'' + 4y' + 4y = 0 \qquad y(-1) = 2, \quad y'(-1) = 1.$$

13 Homework 13

Section 3.4 Complex roots of the Characteristic Equation 1, 5, 8, 10, 17, 21 (might enjoy 28, 33)

3.4.1 Use Euler's formula to write $\exp(1+2i)$ in the form $a+ib$.

3.4.5 Use Euler's formula to write 2^{1-i} in the form $a+bi$.

3.4.8 Find the general solution to

$$y'' - 2y' + 6y = 0$$

3.4.10 Find the general solution to

$$y'' + 2y' + 2y = 0$$

3.4.17 Find the solution to the initial value problem

$$y'' + 4y = 0 \qquad y(0) = 0, \quad y'(0) = 1$$

3.4.21 Find the solution to the initial value problem

$$y'' + y' + 1.25y = 0 \qquad y(0) = 3, \quad y'(0) = 1$$

3.4.28 In this problem we outline a differential derivation of Euler's formula.

a. Show that $y_1(t) = \cos t$ and $y_2(t) = \sin t$ are a fundamental set of solutions of $y'' + y = 0$; that is, show that they are solutions and their Wronskian is not zero.

b. Show (formally) that $y = e^{it}$ is also a solution of $y'' + y = 0$. Therefore,

$$e^{it} = c_1 \cos t + c_2 \sin t \text{ equation } *$$

for some contants c_1 and c_2. Why is this so?

c. Set $t = 0$, to show $c_1 = 1$

d. Differentiate equation * and then set $t = 0$ to conclude that $c_2 = i$. Use c_1, c_2 in equation * to arrive at Euler's formula.

3.4.33 If the functions y_1 and y_2 are linearly independent solution of

$$y'' + p(t)y + q(t)y = 0,$$

show that between consecutive zeros of y_1 there is one and only one zero of y_2. Note that this result is illustrated by the solutions of $y_1 = \cos t$ and $y_2 = \sin t$ of the equation $y'' + y = 0$.

Hint: Suppose that t_1 and t_2 are two zeros of y_1 between which that are no zero's of y_2. Apply Rolle's theorem to y_1/y_2 to reach a contradiction.

14 Homework 14

Section 3.5 Reduction of order 23, 28

3.5.23 Use the method of reduction of order to find a second solution to

$$t^2y'' - 4ty' + 6y = 0, \; t > 0; \qquad y_1(t) = t^2$$

3.5.28 Use the method of reduction of order to find a second solution to

$$(x-1)y'' - xy' + 4x^3y = 0, \; x > 1; \qquad y_1(x) = e^x$$

15 Homework 15

Section 3.6 Method of Undetermined Coefficients 1, 6, 9, 10, 13 (algebra challenge 18)

3.6.1 find the general solution to

$$y'' - 2y' - 3y = 3e^{2t}$$

3.6.6 find the general solution to

$$y'' + 2y' + y = 2e^{-t}$$

3.6.9 find the general solution to

$$u'' + \omega_o^2 u = \cos \omega t \quad \omega^2 \neq \omega_0^2.$$

3.6.10 find the general solution to

$$u'' + \omega_o^2 u = \cos \omega_o t.$$

3.6.13 Find the solution to the inital value problem

$$y'' + y' - 2y = 2t, \qquad y(0) = 0, \quad y'(0) = 1$$

3.6.18 Find the solution to the inital value problem

$$y'' + 2y' + 5y = 4e^{-t} \cos 2t, \qquad y(0) = 1, \quad y'(0) = 0$$

16 Homework 16

Section 3.7 Variation of Parameters 1, 4, 10, 13, 19 (remember $[\ln(\sec(t) + \tan(t))]' = 1/\cos(t)$

3.7.1 Use the method of variation of parameters to find a particular solution of the given differential equation.

$$y'' - 5y' + 6y = 2e^t$$

Then check your answer by using the method of undetermined coefficients.

3.7.4 Use the method of variation of parameters to find a particular solution of the given differential equation.

$$4y'' - 4y' + y = 16e^{t/2}$$

Then check your answer by using the method of undetermined coefficients.

3.7.10 Find the general solution of the differential equation:

$$y'' - 2y' + y = e^t/(1 + t^2)$$

.

3.7.13 Verify that y_1 and y_2 satisfy the corresponding homogeneous equation; then find a particular solution of the given nonhomogeneour equation:

$$t^2 y'' - 2y = 3t^2 - 1, \quad t > 0; \qquad y_1(t) = t^2, \quad y_2(t) = t^{-1}.$$

3.7.19 Verify that y_1 and y_2 satisfy the corresponding homogeneous equation; then find a particular solution of the given nonhomogeneour equation, g is an arbitrary continuous function.

$$(1 - x)y'' + xy' - y = g(x), \quad x > 0; \qquad y_1(x) = e^x, \quad y_2(x) = x.$$

17 Homework 17

Section 3.8 Mechanical Vibrations 1, 6, 11, 24 and lab 2?

3.8.1 Determine $\omega_0, R,$ and δ so as to write the given expression in the form $u = R\cos(\omega_0 t - \delta)$:

$$u = 3\cos 2t + 4\sin 2t.$$

3.8.6 A mass weighing 2 lb stretches a spring 6in. If the mass is pulled down an additional 3 in. and then released, and if there is no damping, determine the position u of the mass at any time t. Plot u versus t. Find the frequency, period, and amplitude of the motion.

3.8.11 A spring is stretch 10 cm by a force of 3 newtons. A mass of 2 kg is hung from the spring and is also attached to a viscous damper that exerts a force of 3 newtons when the velocity of the mass is 5 m/sec. If the mass is pulled down 5 cm below its equilibrium position and given an initial downward velocity of 10 cm/sec, determine its position u at any time t. Find the quasi frequency μ and the ratio of μ to the natural frequency of the corresponding undamped motion.

3.8.24 The position of a certain spring-mass system satisfies the initial value problem:

$$\frac{3}{2}u'' + ku = 0 \qquad u(0) = 2, \quad u'(0) = \nu.$$

If the period and amplitude of the resulting motion are observed to be π an 3, respectively, determent the values of k and ν.

18 Homework 18

Section 3.9 Forced Vibrations 1, 6, 8abc, 8d (yes part 8d is a separate problem)

3.9.1 Write $\cos 9t - \cos 7t$ as a product of two trigonometric functions of different frequencies.

3.9.6 A mass of 5 kg stretches a spring 10 cm. The mass is acted on by an external forces of $10\sin(t/2)$ N (newtons) and moves in a medium that imparts a viscous force of 2 N when the speed of the mass is 4 cm/sec. If the mass is set in motion from its equilibrium position with an initial velocity of 3 cm/sec, formulate the initial value problem describing the motion of the mass.

3.9.8abc a. Find the solution to hte initial value problem in Problem 6

b. Identify the transient and steady-state parts of the solution.

c. Plot the graph of the steady state solution.

3.9.8d If the given external force is replaced by a force on $2\cos\omega t$ of frequency ω, the value of ω for which the amplitude of the forced response is maximum.

19 Homework 19

Section 6.1 Laplace Definition 1, 5, 11, 15

6.1.1 Sketch the graph and determine if f is continuous, piecewise continuous, of neithe on the interval $0 \le t \le 3$

$$f(t) = \begin{cases} t^2, & 0 \le t \le 1 \\ 2+t, & 1 < t \le 2 \\ 6-t, & 2 < t \le 3 \end{cases}$$

6.1.5 Find the Laplace transform of each of the following functions.

a. t

b. t^2

c. t^n, where n is a positive integer

6.1.11 Recall $\cos bt = (e^{ibt} + e^{-ibt})/2$ and $\sin bt = (e^{ibt} - e^{-ibt})/2i$. Find the Laplace transform of $\sin bt$.

6.1.15 Using integration by parts, find the Laplace transform of te^{at}, a is a real constant.

20 Homework 20

Section 6.2 Solution of Initial Value Problems 1, 4, 14, 18, 21

6.2.1 Find the inverse Laplace transform of

$$\frac{3}{s^2+4}$$

6.2.4 Find the inverse Laplace transform of

$$\frac{3s}{s^2-s-6}$$

6.2.14 Use the Laplace transform to solve the initial value problem

$$y'' - 4y' + 4y = 0; \qquad y(0) = 1, \quad y'(0) = 1$$

6.2.18 Use the Laplace transform to solve the initial value problem

$$y^{(4)} - y = 0; \qquad y(0) = 1 \quad y'(0) = 0 \quad y''(0) = 1, \quad y'''(0) = 0$$

6.2.21 Use the Laplace transform to solve the initial value problem

$$y'' - 2y' + 2y = \cos t; \qquad y(0) = 1, \quad y'(0) = 0.$$

21 Homework 21

Section 6.3 Step Functions 1, 9, 13, 18, 25, 27

6.3.1 Sketch the graph of the function on the interval $t \geq 0$

$$u_1(t) + 2u_3(t) - 6u_4(t)$$

6.3.9 Find the Laplace transform of

$$f(t) = \begin{cases} 0, & t \leq \pi \\ t - \pi, & \pi \leq t < 2\pi \\ 0, & t \geq 2\pi \end{cases}$$

6.3.13 Find the inverse Laplace transform of the given function

$$F(s) = \frac{3!}{(s-2)^4}$$

6.3.18 Find the inverse Laplace transform of the given function

$$F(x) = \frac{e^{-s} + e^{-2s} - e^{-3s} - e^{-4s}}{s}$$

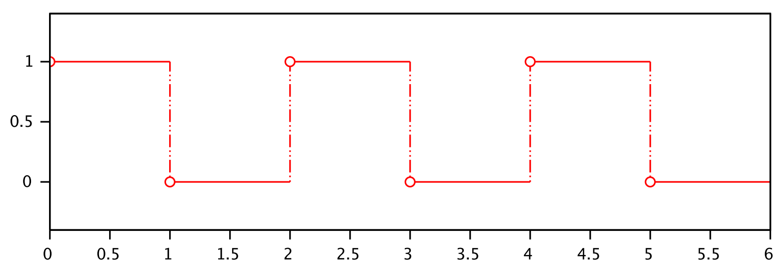
6.3.25 Find the Laplace transform of the given function.

$$f(t) = \begin{cases} 1, & 0 \le t < 1 \\ 0, & 1 \le t < 2 \\ 1, & 2 \le t < 3 \\ 0, & t \ge 3 \end{cases}$$

6.3.27 Find the Laplace transform of the given function. Assume that term-by-term integration of the infinite series is permissible.

$$f(t) = 1 + \sum_{k=1}^{\infty} (-1)^k u_k(t).$$

This is a square wave.



22 Homework 22

Section 6.4 Discontinuous Forcing 3, 11 Section 6.5 Impulse Functions 1, 5, 12 and lab3?

6.4.3 Find the solution to the given initial value problem. Draw the graphs of the solution and of the forcing function; explain how they are related.

$$y'' + 4y = \sin(t) - u_{2\pi}(t)\sin(t - 2\pi); \qquad y(0) = 0, \quad y'(0) = 0.$$

6.4.11 Find the solution to the given initial value problem. Draw the graphs of the solution and of the forcing function; explain how they are related.

$$y'' + 4y = u_{\pi}(t) - u_{3\pi}(t), \qquad y(0) = 0, \quad y'(0) = 0.$$

6.5.1 Find the solution of the initial value problem and draw its graph:

$$y'' + 2y' + 2y = \delta(t - \pi); \qquad y(0) = 1, \quad y'(0) = 0.$$

6.5.5 Find the solution of the initial value problem and draw its graph:

$$y'' + 2y' + 3y = \sin t + \delta(t - 3\pi); \qquad y(0) = 0, \quad y'(0) = 0.$$

6.5.12 Find the solution of the initial value problem and draw its graph:

$$y^{(4)} - y = \delta(t - 1); \qquad y(0) = 0, \quad y'(0) = 0, \quad y''(0) = 0, \quad y'''(0) = 0.$$

23 Homework 23

Section 5.1 Review of Power Series 1, 4, 13, 21

5.1.1 Determine the radius of convergence of

$$\sum_{n=0}^{\infty} (x-3)^n$$

5.1.4 Determine the radius of convergence of

$$\sum_{n=0}^{\infty} 2^n x^n$$

5.1.13 Fine the Taylor series about the point x_0 for $\ln x$, $\quad x_0 = 1$ and determine the radius of convergence of the series.

5.1.21 Rewrite the sum whose generic term involves x^n

$$\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2}.$$

24 Homework 24

Section 5.2 Series Near Ordinary Point I 5, 17, 21

5.2.5 Solve by means of a power series about the given point x_0. Find the recurrence relations; also find the first four terms in each of two linearly independent solutions (unless the series terminates sooner). If possible find the general term in each soluton.

$$(1-x)y'' + y = 0, \quad x_0 = 0$$

5.2.17 Find the first five nonzero terms in the solution of the given initial value problem. Plot the four term and the five term approximations to the solution on the same axes. And from this estimate an interval in which the four term approximation is reasonably accurate.

$$y'' + xy' + 2y = 0, \qquad y(0) = 4, \quad y'(0) = -1$$

5.2.21 The Hermite Equation. The equation

$$y'' - 2xy' + \lambda y = 0, \qquad -\infty < x < \infty,$$

where λ is a constant, is known as the Hermite equation, which is important in mathematical physics.

a. Find the first four terms in each of two linearly independent solutions about $x = 0$.
b. Observe that if λ is a nonnegative even integer, then one or the other of the series solutions terminates and becomes a polynomial. Fine the polynomial solutionsf for $\lambda = 0, 2, 4, 6, 8$ and 10. Note the polynomial is determined up to a multiplicative constant
c. The Hermite polynomial $H_n(x)$ is defined as the polynomial solution of the Hermite equation with $\lambda = 2n$ and for which the coeefficent of x^n is 2^n. Find $H_0(x), \ldots H_5(x)$.

25 Homework 25

Section 5.3 Series Near Ordinary Point II 1, 6, 8, 10c, 11

5.3.1 If $y = \phi(x)$ is a solution of the the initil value problem

$$y'' + xy' + y = 0, \qquad y(0) = 1 \quad y'(0) = 0,$$

determinte $\phi''(x_0), \phi'''(x_0)$ and $\phi^{(4)}(x_0)$

5.3.6 Determine a a lower bound for the radius of convergence of the series solution about each given x_0 for the differential equation

$$(x^2 - 2x - 3)y'' + xy' + 4y = 0; \qquad x_0 = 4, \quad x_0 = -4, \quad x_0 = 0$$

5.3.8 Determine a a lower bound for the radius of convergence of the series solution about each given x_0 for the differential equation

$$xy'' + y = 0; \qquad x_0 = 1.$$

5.3.10c Chebyshev Equation. The Chevyshev differential equation is

$$(1 - x^2)y'' - xy' + \alpha^2 y = 0$$

where α is a constant. Find a polynomial solution for each of the cases $\alpha = n = 0, 1, 2, 3$.

5.3.11 Find the first four nonzero terms in each of two linearly independent power series solutions about the origin. What do you expect the radius of convergence to be for each solution?

$$y'' + (\sin x)y = 0$$

26 Homework 26

Section 5.4 Regular Singular Points 1, 4, 6, 17

5.4.1 Find all singular points of the equation and determine whether each one is regular or irregular:

$$xy'' + (1 - x)y' + xy = 0$$

5.4.4 Find all singular points of the equation and determine whether each one is regular or irregular

$$x^2(1 - x^2)y'' + (2/x)y' + 4y = 0$$

5.4.6 Find all singular points of the equation and determine whether each one is regular or irregular

$$x^2y'' + xy' + (x^2 - \nu^2)y = 0, \qquad \text{Bessel equation}$$

5.4.17 Find all singular points of the equation and determine whether each one is regular or irregular

$$(\sin x)y'' + xy' + 4y = 0$$

27 Homework 27

Section 5.5 Euler Equations 1, 2, 4, 5, 18

5.5.1 Determine the general solution of the differential equation that is valid in any interval not including the singular point:

$$x^2y'' + 4xy' + 2y = 0$$

5.5.2 Determine the general solution of the differential equation that is valid in any interval not including the singular point:

$$(x+1)^2y'' + 3(x+1)y' + 0.75y = 0$$

5.5.4 Determine the general solution of the differential equation that is valid in any interval not including the singular point:

$$x^2y'' + 3xy' + 5y = 0$$

5.5.5 Determine the general solution of the differential equation that is valid in any interval not including the singular point:

$$x^2y'' - xy' + y = 0$$

5.5.18 Find all values of β for which all solution of

$$x^2y'' + \beta y = 0$$

approach zero as $x \to 0$.

28 Extra problems on Laplace Transforms and Series

Extra Laplace Problems Extra Series Problems