Graph Theory Homework Summer 2018

Based on Gross and Yellen 2th Edition

July 23, 2018

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These are problems will be due both daily and at the end of classes. This PDF file was created on July 23, 2018.

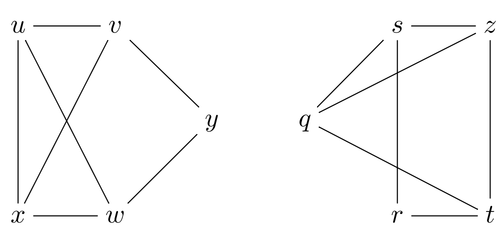
1 Homework 01

Section 2.1 Iso's 2.1.4, 2.1.8, 2.1.17 and 2.1.18 and old quiz 1 from Summer 06:
`https://www.math.fsu.edu/~bellenot/class/su09/graph/oldq1.pdf`

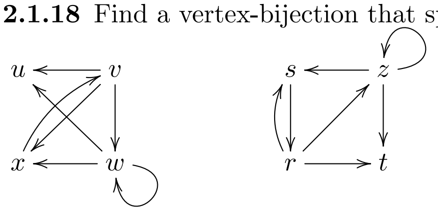
2.1.4 Find all possible isomorphisms tyepes of the given kind of simple graph: A 6-vertex tree.

2.1.8 Find all possible isomorphisms tyepes of the given kind of simple digraph: A simple 4-vertex digraph with exactly four arcs.

2.1.17 Find a vertex-bijection that specifies and isomorphism between the two graphs:

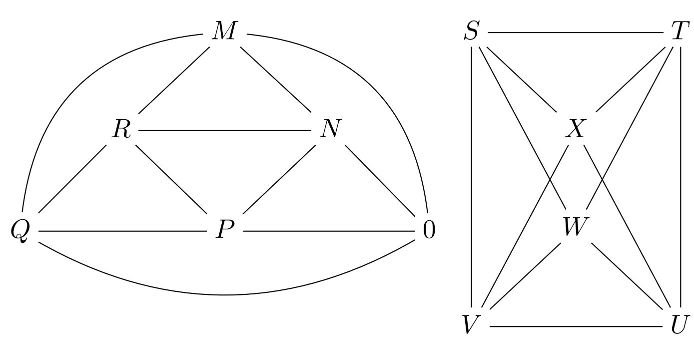


2.1.18 Find a vertex-bijection that specifies and isomorphism between the two digraphs:



oldq1.1 List all possible isomorphism types of trees with 6 edges carefully so that no two trees in your list are isomorphic.

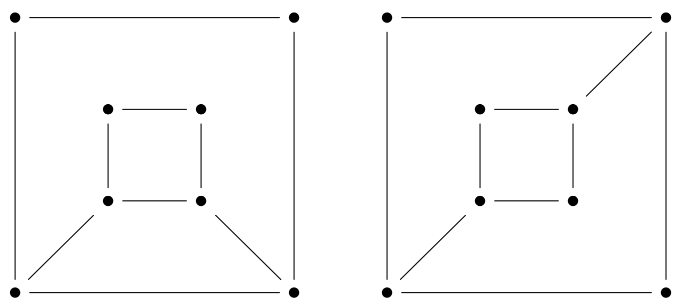
oldq1.2 For the pair of graphs below, decide if they are isomorphic or not. If they are isomorphic, then give an isomorphism. If they are not isomorphic, then explain why they are not isomorphic.



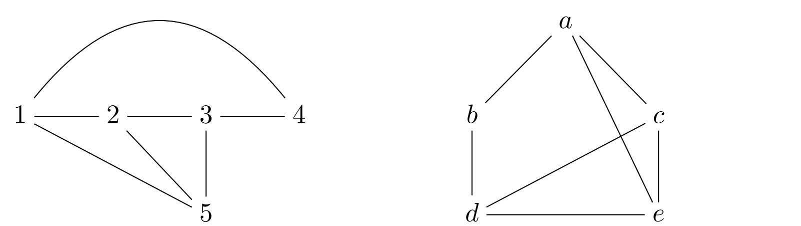
2 Homework 02

Section 2.5 Iso's: 2.5.3, 2.5.5, 2.5.6, 2.5.25

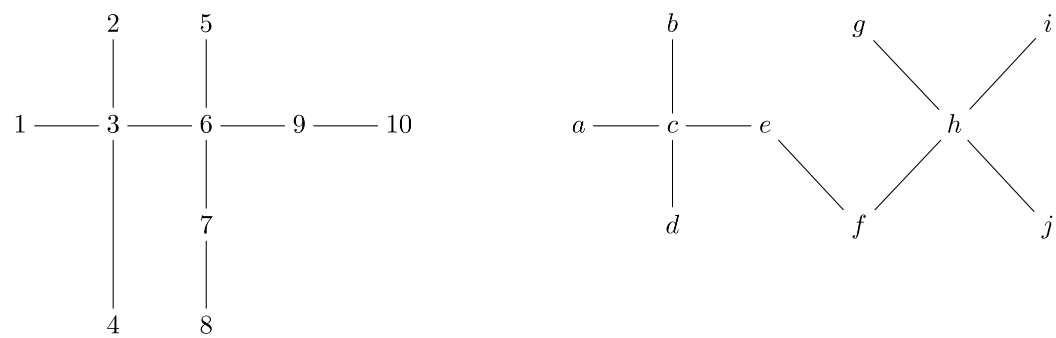
2.5.3 Explain why the graphs are not isomorphic.



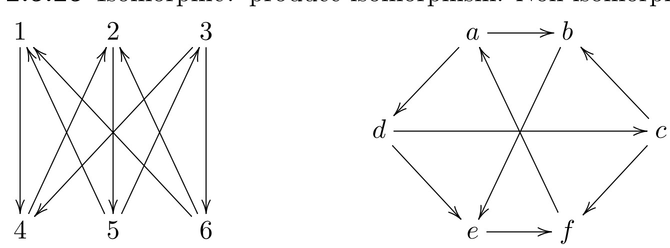
2.5.5 Isomorphic? produce isomorphism. Non-isomorphic? Explain.



2.5.6 Isomorphic? produce isomorphism. Non-isomorphic? explain



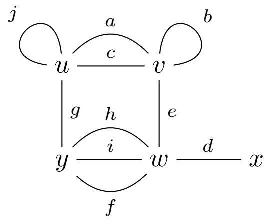
2.5.25 Isomorphic? produce isomorphism. Non-isomorphic? explain why not.



3 Homework 03

Section 2.6 Matrix: 2.6.2, 2.6.7, 2.6.15, 2.6.17

2.6.2 Find the adjacency matrix, A_G, the incident matrix I_G and the table $I_{V,E}(G)$ For G below:



2.6.7 Draw the graph with the adjacency matrix A. The vertices are in the order a, b, c, d

$$A = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

2.6.15 Draw the digraph with the adjacency matrix A. The vertices are in the order a, b, c, d

$$A = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

2.6.17 For the graph with the adjacency matrix A. (The vertices are in the order a, b, c, d)

$$A = \begin{pmatrix} 2 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

Draw the graph G and compute A^2 and show $A^2[a, a]$ is the number of paths of length two in G from a to itself. Show $A^2[d, a]$ does the same for paths of length two from d to a.

4 Homework 04

Sections 1.6 and 1.7 Degrees: 1.6.1 (hint page 262), 1.7.1, 1.7.2, 1.7.3

1.6.1 Formulate the personnel-assignment problem [Application 1.3.1] as a maximum flow problem (Hint: add an artificial source and sink to the bipartite graph) (Hint page 262)

1.7.1 A 20-vertex graph has 62-edges. Every vertex has degree 3 or 7. How many vertices have degree 3?

1.7.2 Either draw a 3-regular 7-vertix graph or prove that none exits.

1.7.3 Prove that no 5-vertex 7-edge simple graph has diameter greater than 2.

5 Homework 05

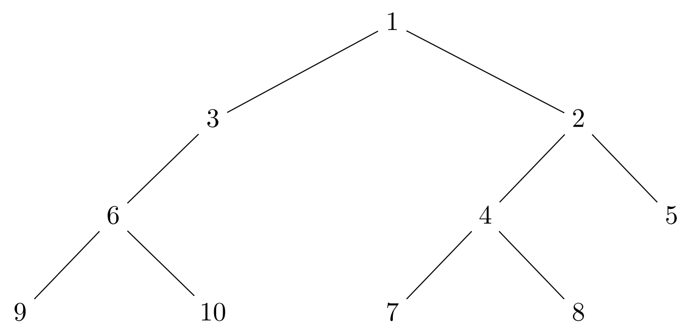
Section 3.1-3 Trees 1: 3.1.7, 3.2.2, 3.2.4, 3.3.2

3.1.7 Either draw the desired graph or explain why no such graph exists: A 9 vertex, 2-component, simple graph with exactly 10 edges and 2 cycles.

3.2.2 Draw all rooted tree types with 5 vertices. How many different graph isomorphism types do they represent?

3.2.4 Draw all possible binary trees of height 3 whose internal vertices have exactly 2 children. Group these into classes of isomorphic rooted trees.

3.3.2 Give level-order, preorder, inorder and postorder tranversals of the binary tree below:



6 Homework 06

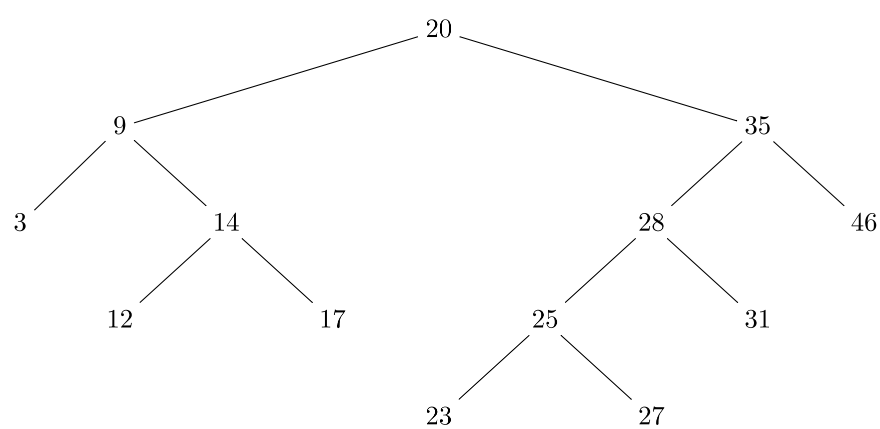
Section 3.3-4 Trees 2: 3.3.8, 3.3.10, 3.4.6, 3.4.12

3.3.8 Represent the given arithmetic expression by an expression tree. Then give the prefix and postfix notations

$$((b \times (c + a) \times (g - d))/(b + d)) \times ((g + c) \times h)$$

3.3.10 Give an example of two 4-vertex binary trees whose level-order and pre-order are a, b, c, d but whose post-order is not.

3.4.6 Using the search tree below, do the operation then draw the search tree. Insert 10, delete 20, insert 15.

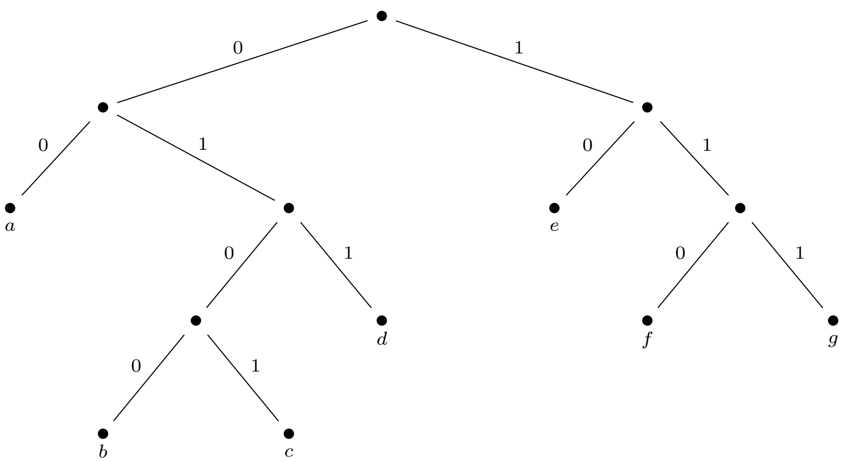


3.4.12 How many different insertion sequences of the keys 1, 4, 9, 19, 41, 49 are there that result in a balanced search tree?

7 Homework 07

Section 3.5-6 Trees 3: 3.5.2, 3.5.5, 3.6.3, 3.6.12 and old quiz 2 from Summer 06:
https://www.math.fsu.edu/~bellenot/class/su09/graph/q2.pdf

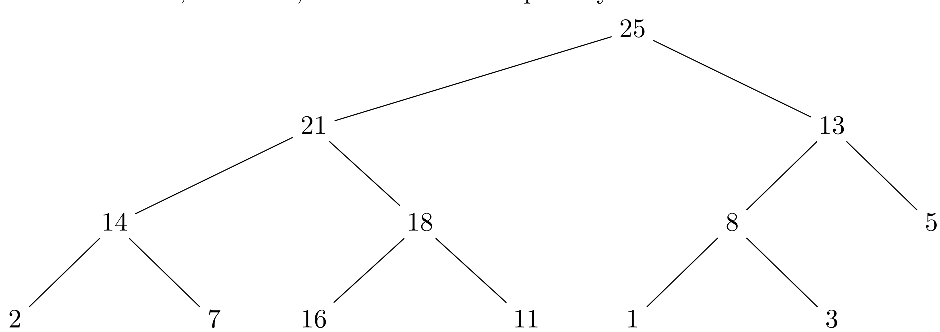
3.5.2 Decode 0100001111110011110 using



3.5.5 Construct a Huffman tree for the given list of symbols and weights. Calculate its average weighted length and encode the words: defaced and baggage. Use left-to-right ordering to break ties.

letter	a	b	c	d	e	f	g	h
frequency	0.1	0.15	0.2	0.17	0.13	0.15	0.05	0.05

3.6.3 Insert 10, insert 15, delete 21 into the priority tree.

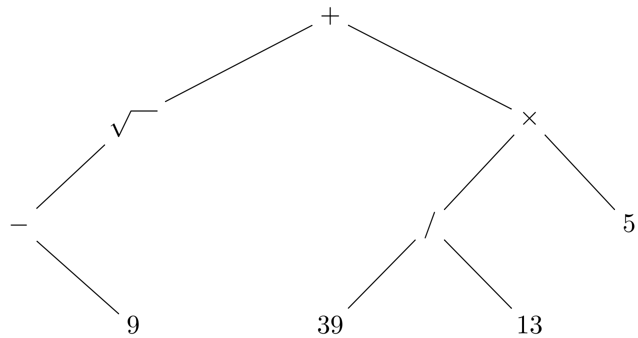


Draw the tree after every operation.

3.6.12 Draw the priority tree for the given sequence or explain why it is not a priority tree:

$$50, 20, 42, 15, 16, 25, 37, 11, 5, 8, 3.$$

oldq2.1 For the expression tree



1. List the vertices in level order
2. List the vertices in preorder
3. List the vertices in inrorder
4. List the vertices in postorder
5. Compute and simplify the complex number given by the tree.

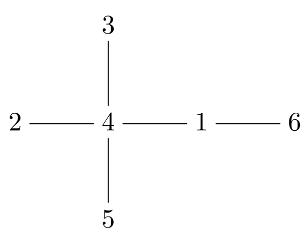
oldq2.2 The complete binary tree of height 2 has 7 vertices.

1. How many insertions sequences (permutations) of 1, 2, 3, 4, 5, 6, 7 when inserted into an initially empty Priority tree result in a complete binary tree?
2. How many insertion sequences (permuations) of 1, 2, 3, 4, 5, 6, 7 when inserted into an initially empty BST (Binary Search Treee) result in a complete binary tree?

8 Homework 08

Section 3.7 and 3.9 Trees 4. 3.7.2, 3.7.8, 3.9.1, 3.9.2

3.7.2 Encode the labeled tree below as a Prüfer sequence



3.7.8 Construct the labeled tree with Prüfer sequence $\{2, 1, 1, 3, 5, 5\}$

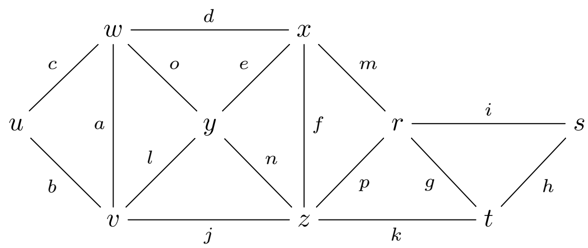
3.9.1 Draw every tree T such that the edge complement $\overline{T}$ is a tree.

3.9.2 What are the minimum and maximum independence numbers of an n-vertex tree?

9 Homework 09

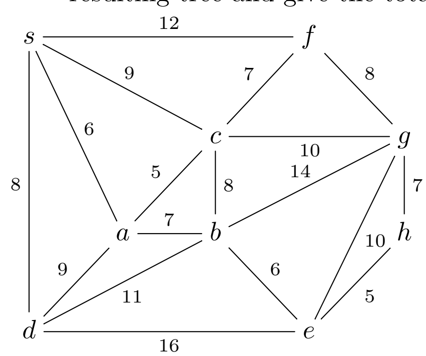
Spanning Trees: 4.2.4, 4.2.11, 4.3.4, 4.3.8 and the online assignments from (with user your fsuid and password your empid:)
https://mobius.math.fsu.edu:8080/mapleta/login/login.do

4.2.4 Start from vertex y, do a depth-first search using Algorithm 4.2.1 including the dfs numbers. First use lexicographical order as the default priority, then repeat using reverse lexicographical order. Use the graph below.



4.2.11 Start from vertex y, do a breadth-first search using Algorithm 4.2.2 including the discovery numbers. First use lexicographical order as the default priority, then repeat using reverse lexicographical order. Use the graph above.

4.3.4 Apply Prim's Algorithm to the weighted graph below to starting with the vertex s and resolving ties like in Example 4.3.1, lexicographic order first by non-tree vertex, then by tree vertex. Draw the resulting tree and give the total weight.

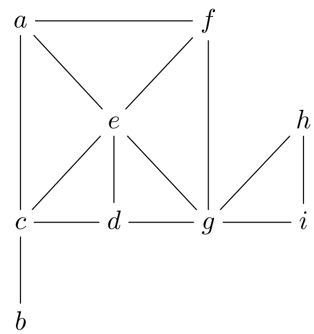


4.3.8 Apply Dijkstra's Algorithm to the weighted graph above to starting with the vertex s and resolving ties like in Example 4.3.1, lexicographic order first by non-tree vertex, then by tree vertex. Draw the resulting tree and give the total weight.

10 Homework 10

DFS applications: 4.4.3, 4.4.4, "Find the blocks and cut vertices of the graph in 4.4.3", 4.4.9

4.4.3 Give the DFS number and the low number for each vertex for a depth-first search of the graph below, starting from vertex c. Use alphabetical order as the default priority. Verify a non-root vertex u is a cut-vertex, if and only if, u has a child w so that $low(w) \geq dfsNumber(u)$.



4.4.4 The same graph above starting from vertex e

Blocks and Cut vertices Find the blocks and cut-vertices of the graph in [4.4.3]

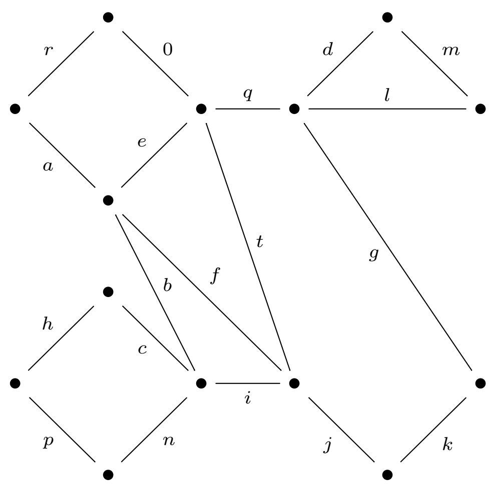
4.4.9 A connected simple graph G with more than 2 vertices with mimimum degree δ then G contains a cycle of length greater than δ.
Show this is not true for non-simple graphs.

11 Homework 11

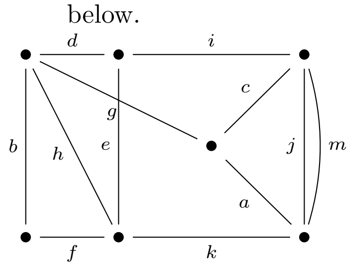
Eulerian Travels: 6.1.3, 6.1.8, 6.1.12, 6.1.16 and the online assignments with user you fsuid and password your empid:
https://mobius.math.fsu.edu:8080/mapleta/login/login.do
http://mobius.math.fsu.edu:8080/mapleta/login/login.do

6.1.3 Which wheels W_n are Eulerian.

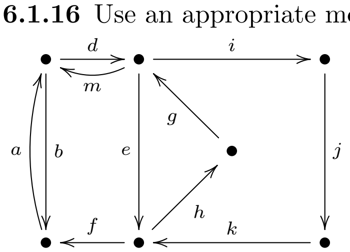
6.1.8 Apply Algorithm,6.1.1 to find an Eulerian tour in the graph below



6.1.12 Use the modified Algorihm 6.1.1 from Exercise 6.1.10 to contruct an open Eulerian trail in graph below.



6.1.16 Use an appropriate modification of Algorithm 6.1.1 to find an Eulerian tour in the digraph below



12 Homework 12

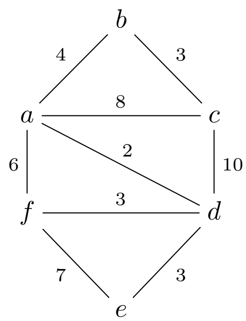
Section 6.2 Eulerian Applications: 6.2.2, 6.2,3, 6.2,12, 6.2.21 and problems 3 and 6 on test1 from Summer 06

https://www.math.fsu.edu/~bellenot/class/su06/graph/test1.pdf

6.2.2 Draw the $(2, 3)$-deBruijn digraph and use it to construct two different $(2, 3)$-deBruijn sequences.

6.2.3 Which vertices of the deBruijn digraphs have self-loops. Justify your answer.

6.2.12 Use Algorithm 6.2.2 to find a minimum-weight postman tour for the weighted graph below



6.2.21 Use the methos of Application 6.2.5 to find an RNA chain whose G and UC fragments are as given

$$G-\text{fragments:}\qquad CCG, G, UCCG, AAAG.$$

$$UC-\text{fragments:}\qquad GGAAAG, GU, C, C, C, C.$$

test1.3 How many edges? does the simple graph G have if

1. Compute the number.
   (a) G has 40 vertices and all of degree 5.
   (b) G is acyclic, has 5 components and 50 vertices
   (c) G is a complete binary tree with height 4.
2. Compute the number and draw a graph, if G has 8 vertices and every vertex has degree 3 or 7 (Give all possibilities.)

test1.6 True or False.

1. Every walk is a trail.
2. If G is connected and $|E| = |V|$, then it has a cycle edge e so that $G - e$ is a tree.
3. An acyclic simple graph has $|E| - |V|$ connected components.
4. For $n \geq 4$, K_n is Eulerian $\iff$ n is odd.
5. Every connected simple graph with a cut edge has a cut vertex.
6. A simple graph G with 40 edges and 20 vertices can have $\Delta(G) \leq 3$.
7. If $n \geq 2$ and $m \geq 2$, the graph $K_{n,m}$ has $m + n$ vertices, mn edges, diameter 2 and no cut edges.
8. A connected graph with degree sequence (5 3 2 2 2 1 1 1 1 1 1) is a tree.
9. For each $h \geq 1$ there is a binary tree of height h so that for each non-leaf vertex v, the height of the left subtree of v is one plus the height of the right subtree of v. (Count empty subtrees as having height -1.)
10. There are 4 isomorphism types of 2-arc 4-vertex simple (loop free) digraphs.

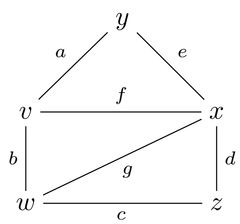
13 Test 1

No homework.

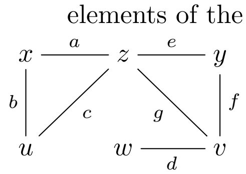
14 Homework 13

Cycle Space: 4.5.2 4.6.2, 4.6.3, 4.6.7.

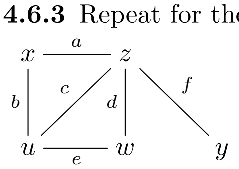
4.5.2 Consider the tree T with edges $\{a, e, g, d\}$ in the graph G below. Find the fundemental system of cycles associated with T. Find the fundmental system of edges-cut associated with T.



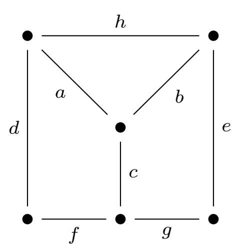
4.6.2 Find the non-null elements of the cycle space $W_C(G)$ for the graph G below. Find the non-null elements of the edge-cut space $W_S(G)$.



4.6.3 Repeat for the graph below:



4.6.7 Show the collection $\{\{a, cd, f\}, \{b, c, e, g\}, \{a, b, h\}\}$ of edge subsets of E_G forms a basis for the cycle space $W_C(G)$ for the graph G below. Find a different basis by choosing some spanning tree and using the associated fundemental system of cycles.



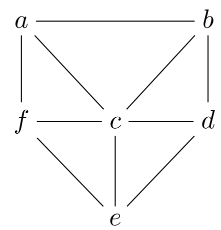
15 Homework 14

Connectivity: 5.1.3, 5.1.4, 5.1.6, 5.1.16.

5.1.3 Either draw the graph or show no such graph exits: A connected graph with 11 vertices and 10 edges and no cut-vertices

5.1.4 Either draw the graph or show no such graph exits: A 3-connected graph with exactly one bridge.

5.1.6 Determine the vertex and edge connectivity of the graph below



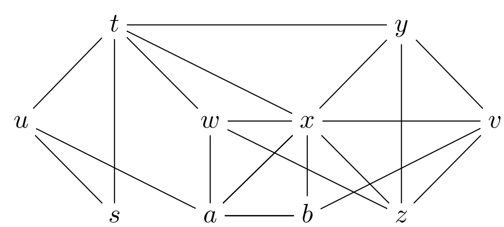
5.1.16 Give an example of a graph G with

$$\kappa_\nu(G) < \kappa_e(G) = \delta(G)$$

16 Homework 15

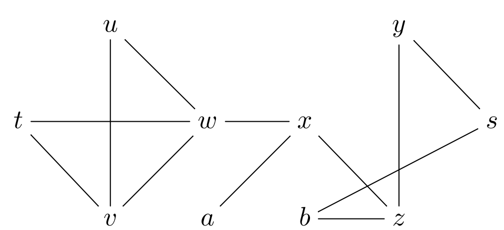
Duality: 5.3.2, 5.3.6, 5.4.2, 5.4.6.

5.3.2 Find the number of internally dijoint uv-paths and use the Certificate of OPtimality to justify your answer in the graph below:



5.3.6 Find the number of internally dijoint uv-paths and use the Certificate of OPtimality to justify your answer in the graph above:

5.4.2 Identify the blocks and draw the block graph for the graph below:



5.4.6 Find two non-isomorphic connected graphs with six verticw, six edges and three blocks.

17 Homework 16

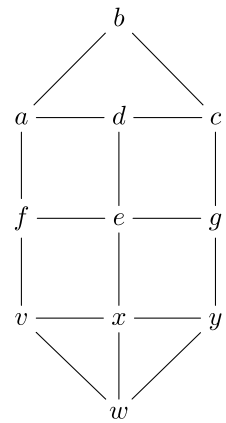
HAM: 6.3.3, 6.3.[6,7] (both 6 and 7 counts as one problem) 6.3.[15,17] (two as one) Gray code of order 5.

6.3.3 Which n-vertex wheel graphs, W_n are Hamiltonian

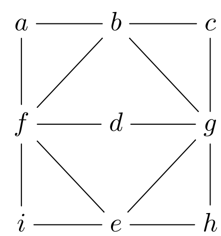
6.3.6 and 7 Draw the graph or prove none exists:

- An 8-vertex simple graph with more than 8 edges that is both Eulerian and Hamiltonian.
- An 8-vertex simple graph with more than 8 edges that is Eulerian but not Hamiltonian.

6.3.15 and 17 Either construct a Hamiltionian cycle or prove none exists.



6.3.15



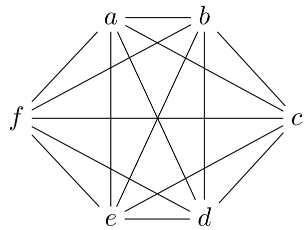
6.3.17

Gray code of order 5 Construct the Gray code of order 5.

18 Homework 17

TSP: Apply Algorihtms 6.4.2 and 6.4.3 to the graph of problems 6.4.4 and 6.4.5.

6.4.4 Apply Algorithm 6.4.2 to the graph below with cost matrix A



$$A = \begin{bmatrix} 0 & 6 & 4 & 6 & 8 & 9 \\ 6 & 0 & 7 & 5 & 7 & 11 \\ 4 & 7 & 0 & 11 & 5 & 8 \\ 6 & 5 & 11 & 0 & 2 & 10 \\ 8 & 7 & 5 & 2 & 0 & 3 \\ 9 & 11 & 8 & 10 & 3 & 0 \end{bmatrix}$$

6.4.5 Apply Algorithm 6.4.2 to the graph above with the cost matrix

$$A = \begin{bmatrix} 0 & 7 & 4 & 9 & 11 & 3 \\ 7 & 0 & 3 & 5 & 2 & 4 \\ 4 & 3 & 0 & 13 & 6 & 8 \\ 9 & 5 & 13 & 0 & 14 & 12 \\ 11 & 2 & 6 & 14 & 0 & 5 \\ 3 & 4 & 8 & 12 & 5 & 0 \end{bmatrix}$$

6.4.3 to 6.4.4 Apply Algorithm 6.4.3 to the graph of 6.4.4.

6.4.3 to 6.4.5 Apply Algorithm 6.4.3 to the graph of 6.4.5.

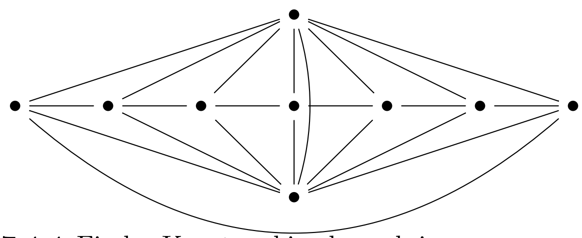
19 Project 1

No extra homework.

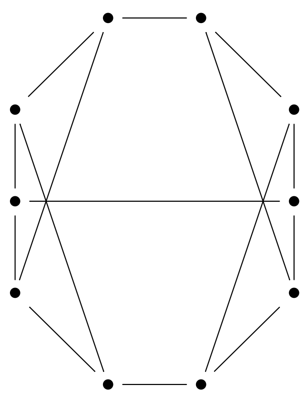
20 Homework 18

Forbidden Graphs, Euler and Duality: 7.4.1, 7.4.4, 7.5.9-11odd 7.5.35-37odd.

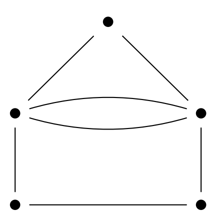
7.4.1 Find a Kuratowski subgraph in

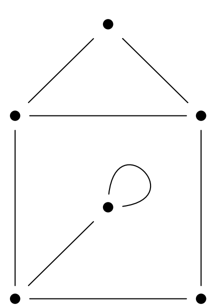


7.4.4 Find a Kuratowski subgraph in



7.5.9-11odd Show $|V| - |E| + |F| = 2$ holds for both graphs below





7.5.35-37odd Either draw a plane graph that meets the description or show none exists

- A bipartite simple graph with 7 vertices and 11 edges.
- A simple planar graph with 144 vertices and 613 edges.

21 Homework 19

Start of digraphs. Critical path: 12.4.2, 12.4.4, 12.6.1

12.4.2 Apply CPM to determine the earliset starting time of each task, the earliest completion time of the entire project and the critical tasks.

Task	a	b	c	d	e	f	g	h
Duration	10	5	3	4	5	6	5	5
Predecessors	-	-	b	a,c	a,c	d	e	f,g

12.4.4 A student must complete 10 courses below before he or she can graduate in applied mathematics. The courses and their prerequisites are listed in the following table. Apply CPM to determine the minimum number of semesters needed to graduate.

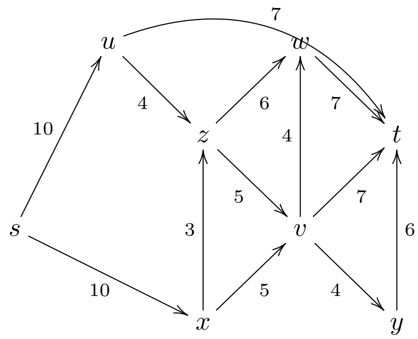
Course	Prerequisites
C1 = Calculus 1	None
C2 = Calculus 2	C1
DM = Discrete Math	C1
C3 = Calculus 3	C2
A = Algorithms	C1,DM
GT = Graph Theory	DM
DE = Differential Equations	C2
S = Statistics	C1
P = Probability	C2,DM
LA = Linear Algebra	DM,C3

12.6.1 Draw all the isomorphisms types of simple digraphs with 4 vertices and 3 arcs.

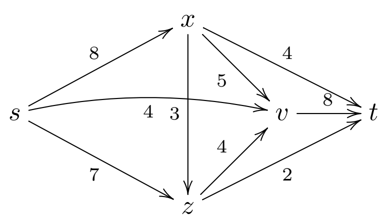
22 Homework 20

Networks 1: 13.1.4, 13.1.6, 13.1.10, 13.1,12

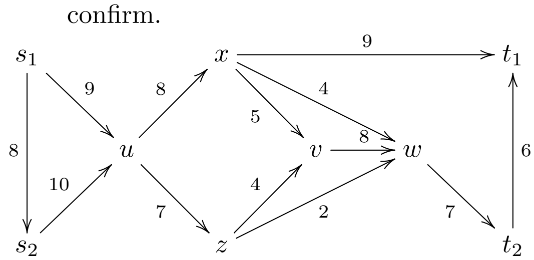
13.1.4 Via trial and error and the Certificate of Optimality to find a maximal flow and a minimum cut.



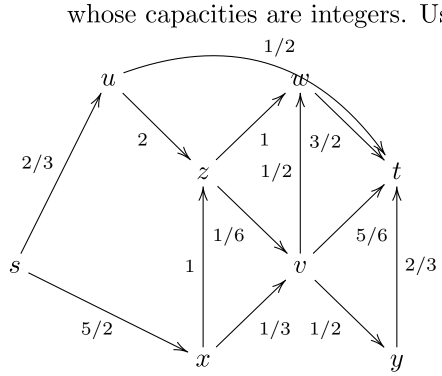
13.1.6 Find all s-t cuts and identify all minimum ones.



13.1.10 Transforma into a single source and sink network and solve. Use the Certificate of Optimality to confirm.



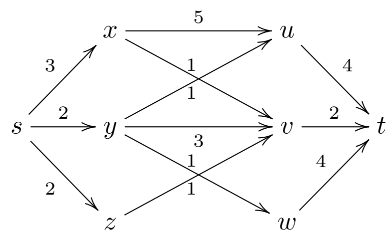
13.1.12 Find a maximal flow and minimal cut in the network below after transforming the network to one whose capacities are integers. Use the Certificate of Optimality to confirm.



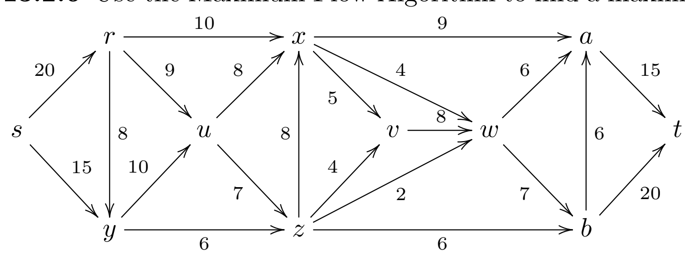
23 Homework 21

Networks 2: 13.2.2, 13.2.6, 13.2.7, 13.2.8 (as network flow problem)

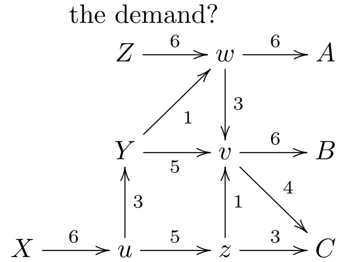
13.2.2 Use the Maximum Flow Algorithm to find a maximum flow and a minimal cut in the network below:



13.2.6 Use the Maximum Flow Algorithm to find a maximum flow and a minimal cut in the network below:



13.2.7 A company maintains three warehouses X, Y and Z, and three stores A, B and C. The warehouses have respectively 500, 500, and 900 mowers in stock. There is an immediate demand from the stores for 700, 600 and 600 mowers repectively.In the graph model below, the arc capacities represent upper bounds on the number of mowers that can be shipped in a single day on that trunk route segment. The immediate nodes may be regarded as transshipment points, where trucks are checked, refueled, maintained etc. Can all the demand be meet? If not, how close can the company come to satisfying the demand?

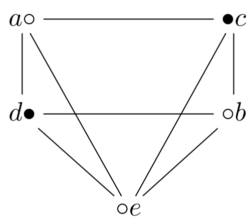


13.2.8 The Johnson, Pate, Shargaa-Sears and Ward families are going to the Winter Park Sidewalk Art Festival. Four cars are available to transport the families to the show. The cars can carry the following numbers of people: car 1, four; car 2, three; car 3 three; and car 4, four. There are four people in each family, and no car can carry more than two people from any family. Formulate the problem of transporting the maximum possible number of people to the festival as a maximum flow problem.

24 Homework 22

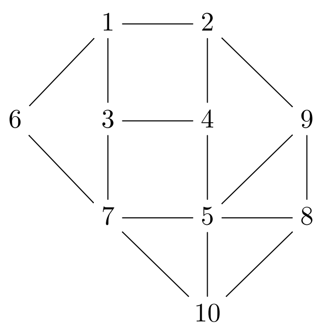
Networks 3: 13.3.2, 13.3.10, 13.4.14, 13.4.18 and the true false problems from Summer 09:
https://www.math.fsu.edu/~bellenot/class/su09/graph/truefalse.pdf

13.3.2 Find the local edge-connectivity between the pair of solid vertices below by finding the maxmimum flow in an appropriate network.



13.3.10 Find the local vertex-connectivity between the pair of solid vertices above by finding the maxmimum flow in an appropriate network.

13.4.14 Find a maximium matching and a minimum vertex cover for



13.4.18 Suppose that there are n workers and n jobs to be preformed. Each worker is qualified to perform exactly k jobs, $k \geq 1$, and each job can be assigned by exactly k workers. Prove that each job can be assigned to a different worker who is qualified for that job.

TFSum09 True or False and a short reason

1. The wheel graph W_n is self dual.
2. The dodecahedron has 12 faces, 30 edges and 20 vertices.
3. The Petersen graph contains a subgraph homeomorphic to K_5.
4. If the connected graph G has $|E| = |V| + 3$ then the cycle space $W_C(G)$ has 15 non-null vectors.
5. A network with a unique maximum flow, has a unique minimum cut.
6. If $n, m \geq 2$ and $n + m \geq 8$ then $K_{n,m}$ is non-planar.
7. For all complete bipartite graphs $\kappa_v(K_{n,m}) = \delta_{\min}(K_{n,m})$
8. Each maximal matching is a maximum matching.
9. For a simple graph G, the minimum number of vertices in a vertex cover of G can be strictly bigger than the maximum number of edges in matching of G.
10. There are 5 isomorphism types of loop-free simple digraphs with 3 vertices and 3 arcs.

25 Test 2

No homework.