MAD 5932 Applied Graph Theory **Test 1** 17 Jul 2009

Show **ALL** work for credit; Give **EXACT** answers when possible; Start each problem on a **SEPARATE** page; Use only **ONE** side of each page; Be neat; Leave margins on the left and top for the **STAPLE**; Calculators can be used for graphing and calculating only; Nothing written on this page will be graded;

1. List all possible isomorphism types of trees with 5 edges carefully so that no two trees in your list are isomorphic.

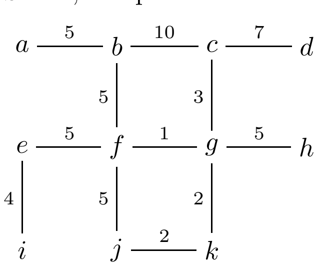
2. For each pair of graphs below, decide if they are isomorphic or not. If they are isomorphic, then give the isomorphism. If they are not isomorphic, then explain why they are not isomorphic.

$$\text{adjacency}(G_1) = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad \text{adjacency}(G_2) = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

$$\text{adjacency}(H_1) = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \qquad \text{adjacency}(H_2) = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

3. How many edges? does the simple graph G have if
   (a) Compute the number.
      i. G has 40 vertices and all of degree 5.
      ii. G is acyclic, has 5 components and 50 vertices
      iii. G is a complete binary tree with height 4.
   (b) Compute the number and draw a graph, if G has 8 vertices and every vertex has degree 3 or 7 (Give all possibilities.)

4. Draw the DFS spanning tree (starting from a) for graph below as ordered rooted tree. For each node in the DFS tree, compute its low number assuming that the dfnumbers start with 0.



5. For the weighted graph above. Remember ties are decided first by the non-tree vertex, then by the tree vertex.
   (a) List the edges (and a running total of tree's weight) as they would be selected by Prim's algorithm to find a minimum weight spanning tree starting at a
   (b) List the edges as they would be selected by Dijkstra's shortest path algorithm to find the shortest path tree that results and the distance from each vertex to a.

There is more test on the other side

6. True or False.

   (a) Every walk is a trail.
   (b) If G is connected and $|E| = |V|$, then it has a cycle edge e so that $G - e$ is a tree.
   (c) An acyclic simple graph has $|E| - |V|$ connected components.
   (d) For $n \geq 4$, K_n is Eulerian $\iff$ n is odd.
   (e) Every connected simple graph with a cut edge has a cut vertex.
   (f) A simple graph G with 40 edges and 20 vertices can have $\Delta(G) \leq 3$.
   (g) If $n \geq 2$ and $m \geq 2$, the graph $K_{n,m}$ has $m + n$ vertices, mn edges, diameter 2 and no cut edges.
   (h) A connected graph with degree sequence (5 3 2 2 2 1 1 1 1 1 1) is a tree.
   (i) For each $h \geq 1$ there is a binary tree of height h so that for each non-leaf vertex v, the height of the left subtree of v is one plus the height of the right subtree of v. (Count empty subtrees as having height -1.)
   (j) There are 4 isomorphism types of 2-arc 4-vertex simple (loop free) digraphs.

7. Prüfer sequences

   (a) Draw the labeled tree corresponding to the Prüfer sequence $\langle 5, 5, 3, 2, 1, 1 \rangle$.
   (b) Encode the tree below as a Prüfer sequence.