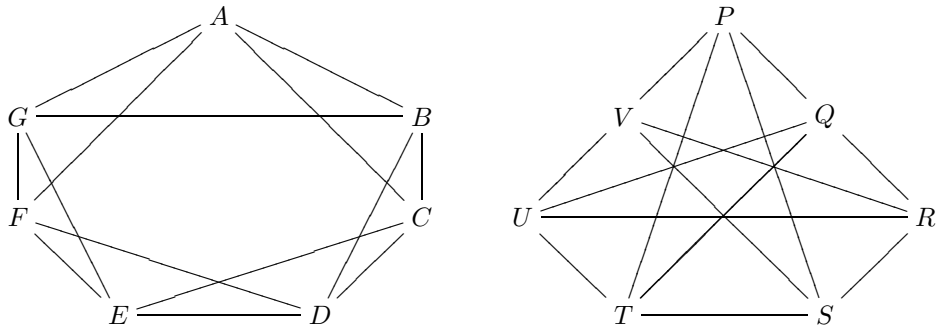
MAD 5932 Applied Graph Theory **Test 1** 26 May 2006

Show **ALL** work for credit; Give **EXACT** answers when possible; Start each problem on a **SEPARATE** page; Use only **ONE** side of each page; Be neat; Leave margins on the left and top for the **STAPLE**; Calculators can be used for graphing and calculating only; Nothing written on this page will be graded;

1. The sequence of numbers in the order 25 55 75 35 45 15 65 are inserted into both a BST (Binary Search Tree) and a Priority Tree. Draw both binary trees. Then, for each tree, delete the value of the root and redraw.

2. For the graphs below, decide if they are isomorphic or not. If they are isomorphic, then give the isomorphism. If they are not isomorphic, then explain why they are not isomorphic.



3. How many vertices does the simple graph G have if

   (a) G has 40 edges and all vertices of degree 5.

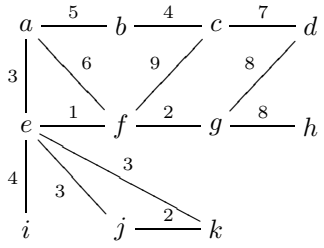
   (b) G is acyclic, has 5 components and 50 edges

   (c) G is tree with 21 leaves and diameter 2.

   (d) G is tree with 11 leaves and diameter 3.

   (e) G has 20 edges and every vertex has degree 3 or 7 (Give all possibilities for the number of vertices.)

4. Draw the DFS spanning tree (starting from a) for graph below as ordered rooted tree. For each node in the DFS tree, compute its low number assuming that the dfnumbers start with 0.



5. For the weighted graph above. Remember ties are decided first by the non-tree vertex, then by the tree vertex.

   (a) List the edges (and a running total of tree's weight) as they would be selected by Prim's algorithm to find a minimum weight spanning tree starting at a

   (b) List the edges as they would be selected by Dijkstra's shortest path algorithm to find the shortest path tree that results and the distance from each vertex to a.

There is more test on the other side

6. True or False.

   (a) Every circuit is a cycle

   (b) If a simple graph G has $|E| = |V|$, then it has a cycle edge e so that $G - e$ is a tree.

   (c) An acyclic graph with $|E| = |V| - 1$ is connected.

   (d) There is a tree with degree sequence (5 3 2 2 2 1 1 1 1 1)

   (e) A simple grap with 40 edges and 20 vertices has a vertex with degree at most 2.

   (f) $\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$ is an adjacency matrix for C_4.

   (g) Starting from the same vertex v and using the same default ordering of vertices in connected graph G, the DFS spanning tree will always have a height at least as large as the height of the BFS spanning tree.

   (h) The diameter of C_n is $\lfloor n/2 \rfloor$ for $n \geq 4$

   (i) There is a binary tree whose vertices in postorder are BAFGDCE and whose vertices in inoder are BEFACGD.

   (j) There are 7 isomorphism types of 2-regular graphs (both simple and non-simple) on 6 vertices.

7. Prüfer sequences

   (a) Draw the labeled tree corresponding to the Prüfer sequence $\langle 2, 1, 1, 3, 5, 5 \rangle$.

   (b) Encode the tree below as a Prüfer sequence.