

1. ASSIGNMENT

This lab continues to explore the connections between graphs of equations of the form

$$ax^2 + 2bxy + cy^2 = 1$$

and properties of the matrix

$$A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

discovered by iteration. You will be given a pair of symmetric matrices, one 2×2 matrix with a 0 eigenvalue and one 3×3 matrix. For each matrix (like A above) on your list you will have to use Scilab to plot (on the same plot) both the conic section (this is described in qform.pdf) and either the arrows like in lambda.pdf. Explicitly your code should find the non-zero L-values and E-vectors by iterations, and compute the polar coordinate or spherical coordinate angles between each E-vectors and the x -axis and z -axis and put the answers in table like Figure 2 or Figure 3. Note that the entries in the table are not rounded off, they have as many digits as scilab gives by default. You can NOT use the built in scilab eigenvalue/eigenvector functions.

2. PARALLEL LINES EXAMPLE

If $A = \begin{bmatrix} 2 & 1 \\ 1 & 0.5 \end{bmatrix}$ then Figure 1 shows the sample plot. Important features of this plot include: 1. the scale in the x and y are the same, (an easy check: the red and blue arrows are perpendicular, do they look perpendicular?); 2. The conic section is pair of parallel lines, (the conic for your second matrix is an ellipsoid in 3 dimensions); 3. Each arrowhead just touches the curve with the exception of the zero eigenvalue whose arrow is roughly 1.5 times the length of the other arrow; 4. The points of contact are critical points nearest, farthest or saddle points.

3. DELIVERABLES

It will be multiple pages which must be stapled in one piece. Paper clips and dog ears will earn a grade of zero. Your lab 5 assignment must be a hardcopy document, no electronic files will be accepted. If you need help, I will look at text or pdf files in email, but no word doc files. Each matrix needs a plot, a table, some code (error free and not filled with iterations). Note that the assignment is due at 3pm Wednesday June 18 and not 5pm nor Friday as usual.

4. HOW THE PARALLEL LINES CASE DIFFERS FROM THE ELLIPTIC CASE

One of the L-values will be zero, which means the matrix will have no inverse. And so the E-vector has to be computed another way (but make sure your E-vector has norm 1). The plotquad.in routine will require an isoview command like in the hyperbolic case. Also you can't divide by Lmin because it is zero. Make your arrow for the Emin roughly 1.5 times the length of the Emax arrow.

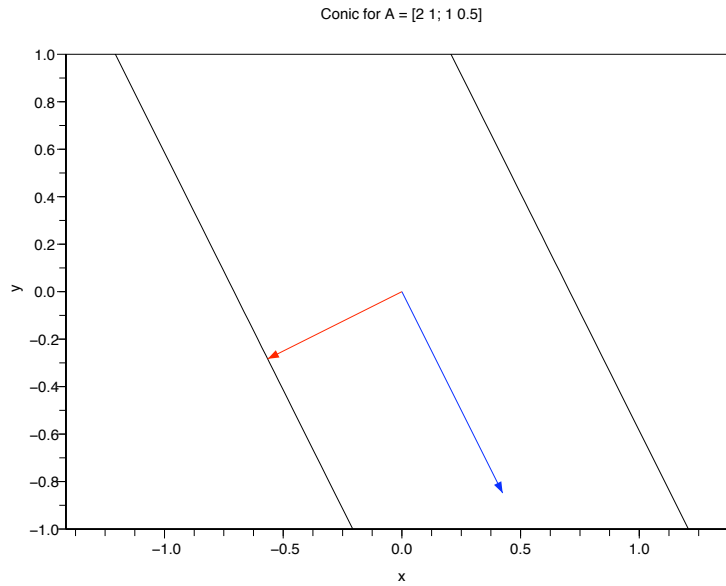


FIGURE 1. $2x^2 + 2xy + 0.5y^2 = 1$ and E-vectors/L-values of $A = [2 \ 1; 1 \ 0.5]$

Ellipse	L-value	E-vector	θ (radians)	θ (degrees)
Max	2.500000	(0.894427, 0.447214)	0.4636476	26.565051
Min	0.0	(0.4472136, -0.8944272)	-1.1071487	-63.434949

FIGURE 2. Table of answers for the parallel lines $A = [2, 1; 1, 0.5]$

Ellipse	L-value	E-vector	θ (degrees)	ϕ (degrees)
Max	4.1160029	(-0.8627822, -0.4646697, 0.1992208)	-149.63042	78.5086
Mid	2.881334	(-0.5055758, 0.7929746, -0.3399770)	120.36958	-19.875476
Min	2.0300772	(2.498D-08, 0.3940474, 0.9190901)	89.999999	23.206575

FIGURE 3. Table of answers for the ellipsoid $A = [3.800413, 0.4949896, -0.2122201; 0.4949896, 3.0157437, -0.4225911; -0.2122201, -0.4225911, 2.2112574]$

5. HOW THE 3×3 ELLIPSOID CASE DIFFERS FROM THE 2×2 CASES

First there are 3 eigenvectors and eigenvalues. The eigenvectors are 3 dimensional so you must use `iterate3d.in` instead of `iterate2d.in`. Finding the middle eigenvalue requires careful reading of the L-vector handout. One plots the ellipsoid with the provided function `sphereplot.in` in the `scilab` directory. See Figure 4.

Ellipsoid eigendirections in blue, equator in black

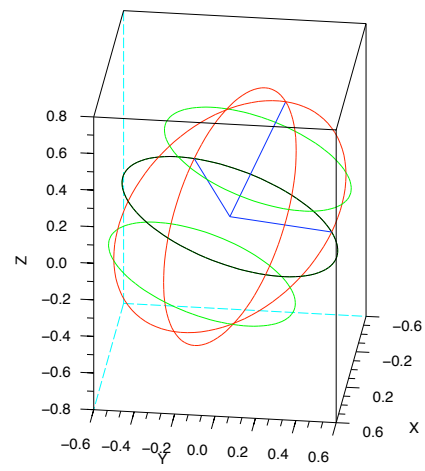


FIGURE 4. Three dimensional ellipsoid