MHF 5206 Foundations **Quiz 4** 6 Jun 2008

Show **ALL** work for credit; Give exact answers when possible. Use a separate page for each of the two problems. Use one side of each page.

1. Let U be a well ordered set and define $f : U \to \mathbb{N}$ and $g : U \to U$ by

$$f(u) = \begin{cases} S(f(v)) & u = S(v) \\ 0 & u = 0 \vee \text{Limit}_U(u) \end{cases} \qquad g(u) = \begin{cases} g(v) & u = S(v) \\ u & u = 0 \vee \text{Limit}_U(u) \end{cases}$$

   [The function f is a sort of mod p, for $p = \omega$ and g is kind of, sort of, a floor or greatest integer analog.]

   (a) Show $\forall u \in U, u = S^{f(u)}(g(u))$ [Remember $S^0(x) = x$ and $S^{n+1}(x) = S(S^n(x))$]

   (b) Show $\pi(u) = (f(u), g(u))$ is an order isomorphism which shows

$$U \leq_o \mathbb{N} \cdot_o \{u \in U \mid u = 0 \vee \text{Limit}_U(u)\}$$

2. True or False and give a short reason why or why not.

   (a) If U and V are well ordered and v is a limit point in V, then $(1, v)$ is a limit point in $U +_o V$.

   (b) $\mathbb{N} +_o \mathbb{N} =_o \mathbb{N}$

   (c) There is a well-ordered set that is order isomorphic to a proper initial seqment of itself.

   (d) Every infinite well-ordered set is order isomorphic to a proper subset of itself.

   (e) The Axiom of Choice says given sets A and B and a relation $P \subseteq A \times B$

$$(\exists x \in A(\forall y \in B\ P(x, y))) \Rightarrow \exists f : A \to B \text{ s.t. } \forall x \in A\ P(x, f(x))$$

   (f) A choice function satisfies $c(A) \in A$ for each set A in its domain.

   (g) In an inductive poset P, chains $\mathscr{C} \subseteq P$ have $\sup \mathscr{C} \in P$

   (h) Every infinite well ordered set has a limit point.

   (i) For non-empty sets A and B, $(A \rightharpoonup B)$ is an inductive poset.

   (j) If $x \in U$ and U is well ordered, then $x \in \text{seq}_U(x)$