MHF 5206 Foundations **Quiz 2** 16 May 2008

Show **ALL** work for credit; Give exact answers when possible. Use a separate page for each of the two problems. Use one side of each page.

1. Let $R, S \subseteq A \times A$ be relations and define

$$RS = \{(a,c) \in A \times A \mid \exists a, b, c \in A \text{ s.t. } (a,b) \in R \ \& \ (b,c) \in S\}$$

and $R^{n+1} = R^n R$; also $R^0 = I = \{(a,a) \mid a \in A\}$ and $R^1 = R$.

   (a) Show $m, n \in \mathbb{N}, R^{m+n} = R^m R^n$

   (b) Define $R^* = \cup_n R^n$ and show R^* is a transitive relation.

2. True or False and give a short reason why or why not.

   (a) For all cardinals $\kappa^{(\lambda+\mu)} = \kappa^\lambda \cdot \kappa^\mu$

   (b) For all cardinals $\kappa + \lambda = \kappa + \mu \Rightarrow \lambda = \mu$

   (c) $(\mathbb{R} \to \mathbb{N}) =_c \mathbb{R}$

   (d) $(A \to (B \to C)) =_c (A \times B \to C)$

   (e) For all $A \neq \emptyset$, $A \leq_c A + A \leq_c A \cdot A \leq_c A^5 \leq_c A^*$

   (f) For all A, $A <_c \mathscr{P}(A)$

   (g) If $\pi : A \to B$ then $a_1 \sim a_2 \iff \pi(a_1) = \pi(a_2)$ is an equivance relation on A

   (h) The usual ordering on $\mathbb{Z}$ is a well-ordering

   (i) If $Sn \leq m$, then $n = m$ or $m \geq n$

   (j) If $f \in (\mathbb{N} \to \mathbb{N})$ and $f(S(n)) = S(f(n))$ then $f(n) = f(0) + n$