$$0, 1, 2, 3, \dots \longrightarrow \omega$$
$$\omega, \omega + 1, \omega + 2, \omega + 3, \dots \longrightarrow 2\omega$$
$$2\omega, 2\omega + 1, 2\omega + 2, 2\omega + 3, \dots \longrightarrow 3\omega$$

MHF 5206-01 Foundations of Mathematics $4\omega \dots \rightarrow \omega^2$

Summer 2008 "B" Session

 $\omega^2, \omega^2 + 1, \omega^2 + 2, \omega^2 + 3 \longrightarrow \omega^2 + \omega$ Instructor: Steven Bellenot bellenot@math.fsu.edu $\omega^2 \text{ http://www.math.fsu.edu/&bellenot/class/su08/found+ 3 \longrightarrow \omega^2 + 2\omega$ MTWDE 11.00.12 22 2212 MTWRF 11:00-12:20 201 LOV

∞ Visit Cantor's Paradise. $\omega^2 + \omega, \omega^2 + 2\omega, \omega^2 + 3\omega, \longrightarrow 2\omega^2$

- ∞ Live the Transfinite life.
- ∞ Avoid being Plagued by Paradox. $\omega^2, 2\omega^2, 3\omega^2, \ldots \longrightarrow \omega^3$

 $\omega, \omega^2, \omega^3, \ldots \longrightarrow \omega^{\omega}$ ∞ Witness truth with respect to an ultrafilter.

- ∞ Non-standard models of \mathbb{N} , (do you really think there is only one model of \mathbb{N} ?) $\omega^{\omega}, 2\omega^{\omega}, 3\omega^{\omega}, \ldots \longrightarrow \omega^{\omega^2}$ only one model of \mathbb{N} ?)
- ∞ Spaces where continous functions must be eventually constant $\omega^{\omega}, \omega^{\omega^2}, \omega^{\omega^3}, \ldots \longrightarrow \omega^{\omega^{\omega}}$
- ∞ Learn why Borel thought general set theory was an illusion

sion $\omega^{\omega}, \omega^{\omega^{\omega}}, \omega^{\omega^{\omega^{\omega}}}, \dots \longrightarrow \epsilon_0$ Perhaps we hype too much. But set theory, as axiomized by Zermelo, is perhaps the only axiom scheme seen in mathematics graduate school. Set theory is important in logic, computer science and analysis as well as serving as one of the basis of mathematics. The course assumes that you can do proofs, especially of naive set theoretical statements.