MAP 3306 eMath2 **Test 2** 6 Jul 2007

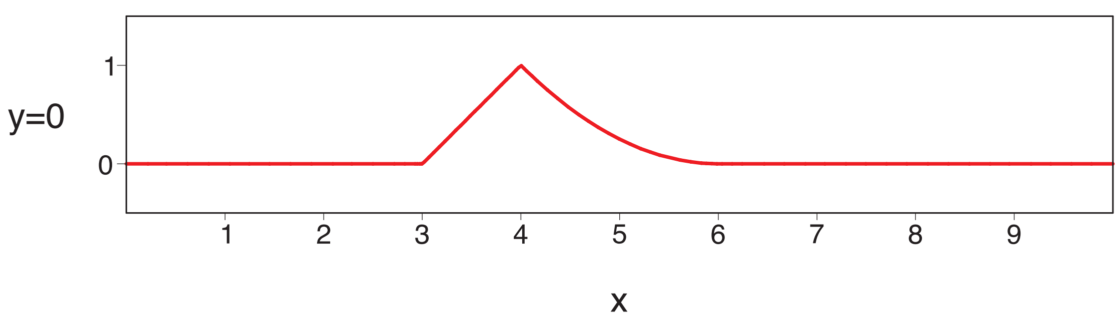
Directions: Show **ALL** work for credit; Give **EXACT** answers when possible; Start each problem on a **SEPARATE** page; Use only **ONE** side of each page; Be neat; Leave margins on the left and top for the **STAPLE**; Nothing written on this page will be graded;

1. Let $f(x) = \sin(\pi x) - 5\sin(3\pi x)$, and $g(x) = \cos(\pi x) - 5\cos(3\pi x)$. Match the PDE problem with its solution. Note there are twice as many solutions (grouped in pairs) than equations. Each pair will be matched once. (We have $c^2 = L = a = b = 1$.)

A. $u_{xx} = u_{tt}$	B. $u_{xx} = u_{tt}$	C. $u_{xx} = u_t$	D. $u_{xx} = u_t$	E. $u_{xx} + u_{tt} = 0$
$u(0,t) = 0$	$u(0,t) = 0$	$u(0,t) = 0$	$u_x(0,t) = 0$	$u(0,t) = 0$
$u(1,t) = 0$	$u(1,t) = 0$	$u(1,t) = 0$	$u_x(1,t) = 0$	$u(1,t) = 0$
$u(x,0) = f(x)$	$u(x,0) = 0$	$u(x,0) = f(x)$	$u(x,0) = g(x)$	$u(x,0) = 0$
$u_t(x,0) = 0$	$u_t(x,0) = f(x)$			$u(x,1) = f(x)$

1a	$u(x,t) = \cos(\pi x)\exp(-\pi^2 t) - 5\cos(3\pi x)\exp(-9\pi^2 t)$
1b	$u(x,t) = \cos(\pi x)\exp(-\pi^2 t) - 5\cos(3\pi x)\exp(-3\pi^2 t)$
2a	$u(x,t) = \sin(\pi x)\exp(-\pi^2 t) - 5\sin(3\pi x)\exp(-3\pi^2 t)$
2b	$u(x,t) = \sin(\pi x)\exp(-\pi^2 t) - 5\sin(3\pi x)\exp(-9\pi^2 t)$
3a	$u(x,t) = \sin(\pi x)\sinh(\pi t)/\sinh(\pi) - 5\sin(3\pi x)\sinh(3\pi t)/\sinh(3\pi)$
3b	$u(x,t) = \sin(\pi x)\sinh(\pi t)/\sinh(\pi) - 5\sin(3\pi x)\sinh(3\pi t)/\sinh(\pi)$
4a	$u(x,t) = \sin(\pi x)\sin(\pi t)/\pi - 5\sin(3\pi x)\sin(3\pi t)/\pi$
4b	$u(x,t) = \sin(\pi x)\sin(\pi t)/\pi - 5\sin(3\pi x)\sin(3\pi t)/3\pi$
5a	$u(x,t) = \sin(\pi x)\cos(\pi t) - 5\sin(3\pi x)\cos(3\pi t)/3$
5b	$u(x,t) = \sin(\pi x)\cos(\pi t) - 5\sin(3\pi x)\cos(3\pi t)$

2. Show $u(x,y) = f(x-y)$ is a solution to the PDE $u_x + u_y = 0$. If $u(x,0)$ looks like the graph below, graph $u(x,2)$.



3. Find the steady state solution of the non-homogeneous problem $u_t = u_{xx} + H$, when the constant $H = 2$, with boundary conditions $u(0,t) = 0$ and $u(1,t) = 5 \neq 0$ (note the non-homogeneous end-point).

4. Find the Fourier integral of $f(x) = \begin{cases} -5x & |x| < 2 \\ 0 & \text{otherwise} \end{cases}$ (Don't forget the Even/Odd Rule.)

There are True/False questions on the other side.

5. True or False and a brief reason why or why not. The following trig identities might be useful

$$\sin A \sin B = [\cos(A-B) - \cos(A+B)]/2$$
$$\sin A \cos B = [\sin(A+B) + \sin(A-B)]/2$$
$$\cos A \cos B = [\cos(A+B) + \cos(A-B)]/2$$

(a) Since $2\sin x \sin nx = \cos[(1-n)x] - \cos[(1+n)x]$ then $\int_0^\pi 2\sin x \sin nx\,dx = 0$ for $n = 1, 2, 3, \ldots$

(b) D'Alembert's solution of the wave equation problem in 1A (above) is $u(x,t) = f(x-t) + f(x+t)$

(c) The standing wave $\sin n\pi x/L$ oscillates at n times the speed of the standing wave $\sin \pi x/L$

(d) $u_{xx} + u_{xy} + u_{yy} = 0$ is parabolic.

(e) $u_{xx} = y u_{yy}$ is hyperbolic for $y > 0$ and elliptic for $y < 0$

(f) The characteristics of $u_{xx} - 5u_{xy} - 6u_{yy} = 0$ are $\Phi = y + x$ and $\Psi = y - 6x$

(g) The characteristics of $u_{xx} + 25u_{yy} = 0$ are $\Phi = y + 5ix$ and $\Psi = y - 5ix$

(h) All solutions of the form $u(x,y) = X(x)Y(y)$ of $u_x + u_y = 0$ are $u = C\exp(k(x-y))$

(i) If $X''(x) = -\omega^2 X(x)$ has the boundary conditions $X(0) = 0$ and $X'(L) = 0$ then $X(x) = C\sin\omega x$ where $\omega L = n\pi/2$ for odd $n = 1, 3, 5, \ldots$

(j) The PDE $u_{xx} + u_x u_y + u_{yy} = 0$ is linear.