MAP 3306 eMath2 **Test 1** 8 Jun 2007

Directions: Show **ALL** work for credit; Give **EXACT** answers when possible; Start each problem on a **SEPARATE** page; Use only **ONE** side of each page; Be neat; Leave margins on the left and top for the **STAPLE**; Nothing written on this page will be graded;

1. Match the function (A) – (E) to its Fourier series (I) – (V):

   A. $f(x) = \frac{\pi^2}{12}(1-3x^2) \quad (-1 < x < 1)$

   B. $f(x) = \begin{cases} \frac{\pi}{4} & 0 < x < \pi \\ -\frac{\pi}{4} & -\pi < x < 0 \end{cases}$

   C. $f(x) = \frac{x}{12}(x^2-\pi^2) \quad (-\pi < x < \pi)$

   D. $f(x) = \frac{\pi^2}{8} - \frac{\pi}{4}|x| \quad (-\pi < x < \pi)$

   E. $f(x) = \begin{cases} \frac{\pi}{2} & 0 < x < \pi \\ 0 & -\pi < x < 0 \end{cases}$

   I. $f(x) = \frac{\pi}{4} + \frac{\sin x}{1} + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \cdots$

   II. $f(x) = \frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \cdots$

   III. $f(x) = \frac{\cos \pi x}{1^2} - \frac{\cos 2\pi x}{2^2} + \frac{\cos 3\pi x}{3^2} + \cdots$

   IV. $f(x) = \frac{\sin x}{1} + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \cdots$

   V. $f(x) = -\frac{\sin x}{1^3} + \frac{\sin 2x}{2^3} - \frac{\sin 3x}{3^3} + \cdots$

2. The function

   $$f(x) = \begin{cases} -\frac{\pi}{4} & \pi/2 < x < \pi \\ \frac{\pi}{4} & -\pi/2 < x < \pi/2 \\ -\frac{\pi}{4} & -\pi < x < -\pi/2 \end{cases}$$

   has Fourier series

   $$f(x) = \frac{\cos x}{1} - \frac{\cos 3x}{3} + \frac{\cos 5x}{5} - \frac{\cos 7x}{7} + \cdots$$

   (a) What statement about a series do we get when we plug in $x = \pi$?

   (b) What statement about a series do we get when we use Parseval's identity?

3. Find the general solution to the PDE $u_x = \frac{u}{x}$ given that $u = u(x, y)$ is function of two variables.

4. Find the Fourier series of $f(x) = -5x \quad (-\pi < x < \pi)$

5. True or False and a brief reason why or why not

   (a) $e^{ib} = \sin b + i \cos b$

   (b) If n is an integer, $e^{in\pi} + e^{-in\pi} = 0$

   (c) The complex fourier series $f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx}$ can be found by using $c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)e^{-inx}\,dx$

   (d) The function $\left(\sin \dfrac{n\pi x}{5}\right)^3$ is 10-periodic.

   (e)

   $$\int \cos \frac{n\pi x}{4}\,dx = \frac{\sin n\pi x}{4n\pi} + C$$

   (f) If both $f(x)$ and $g(x)$ are odd then $f(x)g(x)$ and $f(x) + g(x)$ are both odd.

   (g) If $f(x)$ is odd and $g(x)$ is even then $f(g(x))$ and $g(f(x))$ are both even.

   (h) For the ODE $y'' + 4y = e^{2t}$ the undetermined coefficient guess for the particular solution is $y_{\text{part}}(t) = Ae^{2t}$

   (i) For the ODE $y'' + 4y = \cos 2t$ the undetermined coefficient guess for the particular solution is $y_{\text{part}}(t) = A\cos 2t + B\sin 2t$

   (j) The PDE below is linear

   $$\frac{\partial^2 u}{\partial x^2} + y^2 \frac{\partial^2 u}{\partial y \partial x} = \frac{\partial u}{\partial t} + t^t$$