MAP 3306 eMath2 **Test 0** 11 May 2007

Directions: Show **ALL** work for credit; Give **EXACT** answers when possible; Start each problem on a **SEPARATE** page; Use only **ONE** side of each page; Be neat; Leave margins on the left and top for the **STAPLE**; Nothing written on this page will be graded;

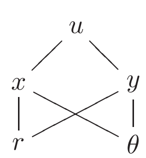
1. Show $u = e^{-t}\sin 5x$ is a solution to
$$\frac{\partial^2 u}{\partial x^2} = 25\frac{\partial u}{\partial t}$$

2. Find the general solution to $y'' - 2y' + 5y = 0$

3. Compute and simplify the integral below assuming n is a positive integer. Your answer should be trig-function-free. Hint: At the end, consider n even and n odd cases separately.
$$\int_0^\pi x\cos nx\,dx$$

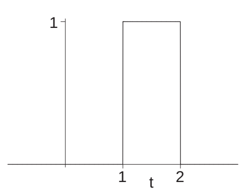
4. A function $u(x, y)$ is also a function of the polar coordinate r and θ via $u(r\cos\theta, r\sin\theta)$. Use the chain rule to show
$$\frac{\partial u}{\partial r}\cos\theta - \frac{1}{r}\frac{\partial u}{\partial \theta}\sin\theta = \frac{\partial u}{\partial x}$$

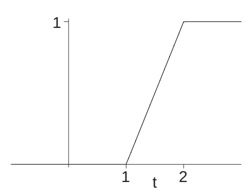
Hint



5. True or False and a brief reason why or why not Here $u(t)$ is the unit step function which is 0 for $t < 0$ and 1 otherwise.

(a) The function graphed below left is $u(t-1) - u(t-3)$





(b) The function graphed above right is $(t-1)u(t-1) - (t-1)u(t-2)$

(c) The trigonometric functions $\sin x$, $\cos x$, and $\tan x$ all have fundamental period 2π.

(d) The following are trig identities $\sin 2x = 2\cos x\sin x$ and $\cos 2x = \cos^2 x - \sin^2 x$

(e) The ODE $y'' + \sqrt{x}y' - 3x^2e^xy = 3x^x$ is linear.

(f) The ODE $(y')^2 + y''' + y^2 = \sin t\cos t$ is second order.

(g) $e^{2x} = \displaystyle\sum_{n=0}^{\infty}\frac{2^nx^n}{n!}$

(h) The radius of converence of $\displaystyle\sum_{n=0}^{\infty}\frac{x^n}{2^n}$ is $\frac{1}{2}$

(i) The gradient of a function $f(x, y, z)$, $\operatorname{grad} f = \nabla f$ is a scalar.

(j) The divergerge of a function $f(x, y, z)$, $\operatorname{div} f$ is a scalar.