MAD 3105 Discrete Math 2 **Test 3** 10 Apr 1996

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper. Start problems on **LEFT** side of the paper only.

1. Big $O$.

A. Prove $O(4n^2 - 5 + 13n^3) = O(100n^3 - n + n^2 - 3)$.

B&C. Arrange in increasing order:

$$O(\log n), O(n!), O(n^6), O(\sqrt{n}), O(3^n), O(n), O(n\sqrt{n}), O(n^n), O(n \log n), O(2^n).$$

2. Determine the successor in lexicographic order.

A. For the permutations $(5, 3, 6, 4, 2, 1)$ and $(2, 3, 1, 6, 5, 4)$.

B. For the five-element subsets of $\{n : 1 \le n \le 9\}$ after each of $\{1, 3, 5, 6, 7\}$, $\{1, 2, 3, 8, 9\}$ and $\{3, 5, 7, 8, 9\}$.

3. & 4. How many of 5 card poker hands are there?

A. With 3 spades and 2 clubs?

B. With 2 Jacks, a five, the three of hearts and a seven?

C. With a full house (3 of a kind and an another pair)?

D. At least one pair?

E. With exactly one Queen and exactly 4 hearts?

5. & 6. How many ways are there of putting 22 balls into 7 boxes,

A. If the balls are distinct?

B. If the balls are identical?

C. If the balls are identical and every box has at least one ball?

D. If the balls are identical and no box has more than 19 balls

E. Prove some box will have at least 4 balls after the balls have been put into the boxes.

7. Give network counterexamples to each statement below.

A. If $|F| = 0$, then every edge has zero flow.

B. If $F(\mathcal{T}, \mathcal{S}) = 0$ and $(\mathcal{S}, \mathcal{T})$ is minimal cut, then $F$ is a maximal flow.

C. If $F(\mathcal{S}, \mathcal{T}) = \text{capacity}(\mathcal{S}, \mathcal{T})$, then $F$ is a maximal flow.

D. If $ab$ is an unsaturated edge with non-zero flow and $(\mathcal{S}, \mathcal{T})$ is a minimal cut, then either both vertices are in $\mathcal{S}$ or both vertices are in $\mathcal{T}$.

8. Use Inclusion-Exclusion for part A&B. (For those who don't know, dice are cube-shaped and thus have 6 faces. On each face there are from one to six dots, representing the numbers one to six. Each die has a face with each number between one and six.) [*Hint* : define the sets $A_i$ so as to count the intersection of the complements of $A_i$.]

A&B. Count the number of ways of rolling 10 distinct dice so that at least one of each of the numbers 1-6 appears.

C. Compute the probability that the above event occurs.