MAD 3105 Discrete Math 2 **Test 2** 6 Mar 1996

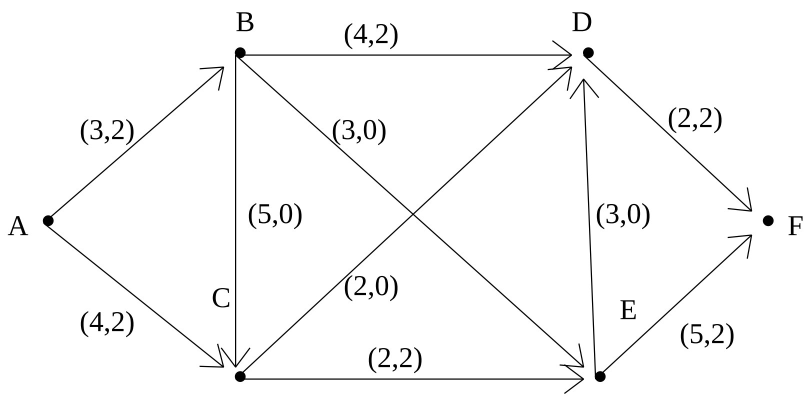
Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper.

1. A lumber company owns 7000 birch trees. Each year the company plans to harvest 12 percent of its trees and then plant 600 new ones. Write a recurrence relation and initial conditions for the number s_n of trees at the end of year n.

2. Given $s_n - s_{n-1} - 6s_{n-2} = 0$. Write the general solution to this recurrence relation. Explicitly find the solution which also satisfies the initial conditions $s_0 = 2, s_1 = 2$.

3. Given $s_n + 3s_{n-1} = 5$. Write the general solution to this recurrence relation. Explicitly find the solution which also satisfies the initial conditions $s_0 = 4$.

4. For the network below, use the flow augmentation algorithm to find a maximal flow, and its value. Use the text's convention for labeling vertices in alphabetical order when there is a choice.



5. Find the smallest sum of an independent set of entries from the matrix below and indicate an independent set of entries that has this smallest sum.

$$\begin{bmatrix} 8 & 2 & 4 & 6 \\ 3 & 1 & 7 & 5 \\ 4 & 6 & 5 & 3 \\ 4 & 5 & 2 & 1 \end{bmatrix}$$

6. Write a recurrence relation and initial conditions for the number s_n of n-bit strings having no four consecutive zeros.

7. Prove by strong induction that the algorithm NPF halts. [NPF computes the number of primes (counting repetitions) in the prime factorization of n.]

```
integer NPF ( integer n )
if n less than or equal to 1
  return 0
else if n is prime
  return 1
else let p be the smallest integer greater than 1 s.t.  p divides n and let q be n/p
  return NPF ( p ) + NPF ( q )
```

8. It can be shown that for an input of length n, the run time, s_n, of some divide and conquer algorithm satisfies the recurrence relation $s_n = 2s_{n/2} + 3n$ and $s_1 = 1$. Solve the recurrence by using the substitution $n = 2^k$.