MAD 3105 Discrete Math 2 **Test 1** 31 Jan 1996

Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper.

1. Tell how many systems of distinct representatives the given sequence of sets has

A. $\{1,2,4\}\{2,4\},\{3\},\{2,3\}$

B. $\{1,4\},\{2\},\{2,3,5\},\{1,2,4\},\{1,2\}$

C. $\{1,2,3,\ldots n\},\{1,2,3,\ldots n\},\{1,2,3,\ldots n\}$

D. $\{1,2,3,\ldots n\},\{n+1,n+2,\ldots n+m\},\{n+m+1,n+m+2,\ldots,n+m+k\}$

2. Binominal coefficients

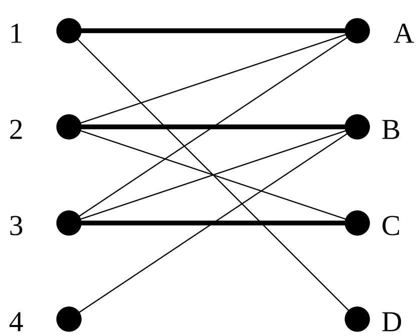
A. Draw Pascal's triangle until you get to the row needed for $(x+y)^7$

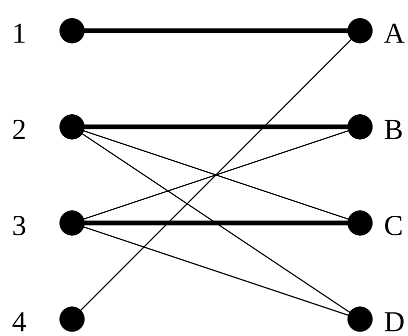
B. Expand $\binom{3n}{3}$ as a polynominal and simplify.

3. Here is a state table with ouput. Draw the transition diagram and list the output for the input sequence 0111100101 assuming A is the initial state.

Input	A	B	C	D	A	B	C	D
0	A	A	B	C	s	d	d	d
1	B	C	D	D	d	d	d	s

4. and 5. There are two bi-partite graphs below each with a matching (independent) set indicated by the bold edges. For each graph, change the graph into a matrix of zero's and one's with the correct one's starred. Procede with the algorithm of section 5.3 carefulling labeling the matrix, until either the end of step 4 or step 6 which ever comes first. If the algorithm stops in step 4, write down the vertices in the minimal cover obtained in step 4. If the algorithm ends in step 6, re-draw the bi-partite graph indicating the new matching.





6. Solve the bottleneck problem below. Show find the minimum completion time and show any smaller time is not a solution.

$$\begin{bmatrix} 3 & 5 & 5 & 3 & 8 \\ 4 & 6 & 4 & 2 & 6 \\ 4 & 6 & 1 & 3 & 6 \\ 3 & 4 & 4 & 6 & 5 \\ 5 & 7 & 3 & 5 & 9 \end{bmatrix}$$

7. Given $s_1 = 5$ and $s_n = 3s_{n-1} - 2^{n-1}$ for $n \geq 2$. Prove by induction $s_n = 3^n + 2^n$ for $n \geq 1$.

8. Devise a finite state machine (show the transition diagram) with inputs $I = \{0,1\}$ which accepts every string but those that contain 4 consecutive inputs (bits) of the form '0101'.