Strong Induction Problems.

```
Boolean BinSearch ( int key, SortedListofIntegers s )
if the length of s is zero
  return false
else if the length of s is one
  return true if the list is key otherwise return false
else let k be the middle element of s, s1 the list before k and s2 the list after.
  if key is k
    return true
  else if key < k
    return BinSearch ( key, s1 )
  else
    return BinSearch ( key, s2 )
```

1. Prove by stong induction on the length of the list s that BinSearch halts.
2. Assuming BinSearch halts, prove by stong induction on the length of the list s that BinSearch correctly determines if key is in the list.

```
List MergeSort ( List s )
if the length of the list is less than or equal 1
  return s
else divide the list into halves s1 and and s2 (or as near halves as possible)
  return Merge( MergeSort(s1), MergeSort(s2) )
```

3. Assuming Merge halts, prove by stong induction on the length of the list s that MergeSort halts.
4. Assuming MergeSort halts and Merge is correct, prove by stong induction on the length of the list s that MergeSort returns a sorted list.

```
VeryLongInteger Multiply ( VeryLongInter x, VeryLongInteger y)
Let n be the number of digits in the longest of x and y, and d = n/2.
If n less than or equal 1
  return x * y
else find x1, x2, y1, y2 each have no more than d digits so that
  x = x1 · 10^d + x2 and y = y1 · 10^d + y2
  let m1 = Multiply ( x1, y1 )
  let m2 = Multiply ( x2, y2 )
  let m3 = Multiply ( x1 + x2, y1 + y2 ) - m1 - m2
  return m1 · 10^(2d) + m3 · 10^d + m2
```

5. Prove by stong induction on the number of digits n that Multiply halts.
6. Assuming Multiply halts, prove by stong induction on the number of digits n that Multiply correctly multiplies the two numbers x and y.