

Additional Subspace Problems

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Subspace Problems: Give a reason why the following sets W are not a subspace. The reason can be one of

- A. The zero vector does not belong to W . (If W is a set of functions, the zero vector is the zero function. If W is a subset of \mathbb{R}^n , the zero vector is the $n \times 1$ vector with all zeros.)
 - B. There is a particular \mathbf{v} in W and scalar c so that $c\mathbf{v}$ is not in W
 - C. There are vectors \mathbf{u} and \mathbf{v} in W so $\mathbf{u} + \mathbf{v}$ is not in W
1. $W = \{\mathbf{u} : \|\mathbf{u}\| = 1\}$
 2. $W = \{c_1\mathbf{u}_1 + c_2\mathbf{u}_2 : c_1 + c_2 = 1\}$ (Hint: there are two cases, are the \mathbf{u}_i vectors independent or dependent.)
 3. $W = \{c_1\mathbf{u}_1 + c_2\mathbf{u}_2 : c_1 \geq 0, c_2 \geq 0\}$ when the \mathbf{u}_i vectors are independent. (The dependent case can go either way.)
 4. For a non-zero vector $\mathbf{v} \in \mathbb{R}^5$, $W = \{\mathbf{u} \in \mathbb{R}^5 : \mathbf{u} \cdot \mathbf{v} = 13\}$
 5. For a non-zero vector $\mathbf{v} \in \mathbb{R}^5$, $W = \{c\mathbf{v} : c \geq 0\}$
 6. For a non-zero vector $\mathbf{v} \in \mathbb{R}^5$, $W = \{c\mathbf{v} : |c| > 0\}$
 7. W is the set continuous functions $f(t)$ with $\int_0^1 f(t) dt = 1$
 8. W is the set continuous functions $f(t)$ with $f(0.2) - f(0.5) = 3$
 9. W is the set continuous functions $f(t)$ that solve the ODE $y'' + y = \sin(2t)$
 10. W is the set continuous functions $f(t)$ that solve the ODE $(y')^2 = 1 + y$

Subspace Problems: show the following are subspaces.

- a. $W = \{\mathbf{u} : \|\mathbf{u}\| = 0\}$
- b. $W = \{c_1\mathbf{u}_1 + c_2\mathbf{u}_2 : c_1 + c_2 = 0\}$
- c. $W = \{c_1\mathbf{u}_1 + c_2\mathbf{u}_2 : c_1 = 0, c_2 = 0\}$
- d. For any vector $\mathbf{v} \in \mathbb{R}^5$, $W = \{\mathbf{u} \in \mathbb{R}^5 : \mathbf{u} \cdot \mathbf{v} = 0\}$
- e. W is the set continuous functions $f(t)$ with $\int_0^1 f(t) dt = 0$
- f. W is the set continuous functions $f(t)$ with $\int_0^1 f(t) \sin(\pi t) dt = 0$
- g. W is the set continuous functions $f(t)$ with $f(0.2) - f(0.5) = 0$
- h. W is the set continuous functions $f(t)$ that solve the ODE $y'' + y = 0$
- i. W is the set continuous functions $f(t)$ that solve the ODE $y'' + e^t y' = \sin(t)y$