1. True or False and a brief reason why or why not. Let A be an $n \times n$ matrix, I the $n \times n$ identity matrix and let B and C be the matrices given below.

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

   (a) If $A - \lambda I$ has a row of zeros, then λ is an eigenvalue of A
   (b) An eigenvector can be zero, but an eigenvalue must be non-zero.
   (c) The vector $X = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}^T$ is an eigenvector for C
   (d) If $Y = \begin{bmatrix} 0 & 1 & 0 & -1 & 0 & 0 \end{bmatrix}^T$, then $C * Y = -Y$.
   (e) B has a characteristic polynomial $p(\lambda)$ which has only the repeated root 1 three times.
   (f) If A is invertible then $\det(A) = 0$
   (g) If $\text{rref}(A)$ is the reduce row echelon form of A, then $\det(A) = \det(\text{rref}(A))$.
   (h) The column vectors of B are a dependent set.
   (i) The vector $X = \begin{bmatrix} 0 & -1 & 3 \end{bmatrix}^T$ is a linear combination of the column vectors of B.
   (j) If $AX = 0$ has a solution, then A is singular.

2. True or False and a brief reason why or why not. Let A be an $n \times n$ matrix, I the $n \times n$ identity matrix and let B and C be the matrices given below.

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1 \end{bmatrix}$$

   (a) $\det(C) = 1$
   (b) If $X = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T$, then $C * X = 2X$.
   (c) The characteristic polynomial of A is $p(\lambda) = \det(A - \lambda I)$
   (d) The eigenvalues of A are the coefficients of the characteristic polynomial of A.
   (e) The numbers $\det(A)$ and $\det(\text{rref}(A))$ are either both zero or both non-zero.
   (f) $B^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$
   (g) If X is an eigenvector for the invertible matrix A, then X is also an eigenvector for A^{-1}.
   (h) If λ is an eigenvalue for A, then λ is also an eigenvalue for $-A$.
   (i) The column vectors of C are linearly dependent.
   (j) If $AX = 0$ has a non-zero solution, then A is singular.

3. True or False and a brief reason why or why not. Let A be an $n \times n$ matrix, I the $n \times n$ identity matrix and let B and C be the matrices given below.

$$B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) $\det(C) = 1$

(b) If $X = \begin{bmatrix} 0 & -1 & 0 & 0 & 1 & 0 \end{bmatrix}^T$, then X is an eigenvector for C.

(c) If $X = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}^T$, then X is an eigenvector for C.

(d) If the matrix product of $B^{-1}D = \begin{bmatrix} -5 & -5 & -5 \\ 0 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}$ then $D = \begin{bmatrix} -5 & -5 & -10 \\ 0 & 4 & 5 \\ 6 & 7 & 14 \end{bmatrix}$. (Hint: **Don't** compute B^{-1}; instead use $(B^{-1})^{-1} = B$)

(e) If $p(s) = \det(A - sI)$ is the characteristic polynomial of the matrix A, then $\det(A)$ is the y-intercept of the graph of $y = p(x)$

(f) The characteric polynomial of $\begin{bmatrix} 5 & -5 \\ 4 & 5 \end{bmatrix}$ is $p(s) = s^2 - 10s + 5$

(g) If X is an eigenvector for the invertible matrix A, then X is also an eigenvector for A^{-1}.

(h) If λ is an eigenvalue for A, then λ^2 is an eigenvalue for A^2.

(i) The column vectors of C are linearly dependent.

(j) The matrix product C^2 is I