LAB8: GEOMETRY MEETS ITERATION PART 4

1. Assignment

This lab continues to explores the connections between graphs of equations of the form

$$ax^2 + 2bxy + cy^2 = 1$$

and properties of the matrix

$$A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

discovered by iteration. We increase the dimension by one and the equation becomes

$$ax^2 + by^2 + cz^2 + 2dxy + 2exz + 2fyz = 1$$

and the matrix is

$$A = \begin{bmatrix} a & d & e \\ d & b & f \\ e & f & c \end{bmatrix}$$

You have been given a symmetric 3×3 matrix. For your matrix ou will have to use Scilab to plot the quadratic surface, which is an ellipsoid, with the provided function sphereplot.in, which is in the usual location for scilab files. Explicitly your code should find the non-zero L-values and E-vectors by iterations, and compute the spherical coordinate angles between each E-vectors and the x-axis and z-axis and put the answers in table like Figure 2. Note
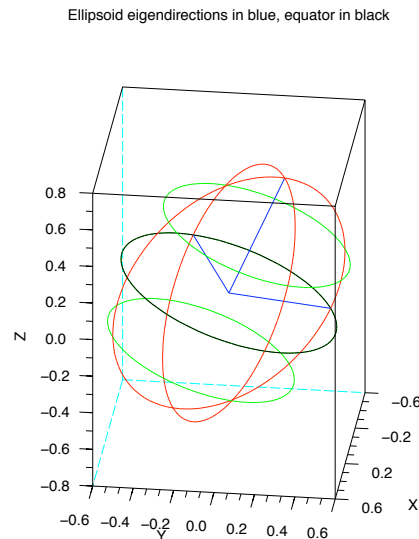

Ellipsoid eigendirections in blue, equator in black

Figure 1. Three dimensional ellipsoid

Ellipse	L-value	E-vector	θ(degrees)	ϕ(degrees)
Max	4.1160029	(0.8627822, 0.4646697, -0.1992208)	28.305631	101.4914
Mid	2.881334	(0.5055758, -0.7929746, 0.3399770)	-57.479688	70.124524
Min	2.0300772	(2.499E-08, 0.3940474, 0.9190901)	89.999999	23.206575

FIGURE 2. Table of answers for the ellipsoid A = $[3.800413, 0.4949896, -0.2122201; 0.4949896, 3.0157437, -0.4225911; -0.2122201, -0.4225911, 2.2112574]$

that the entries in the table are not rounded off, they have as many digits as scilab gives by default. You can NOT use the built in scilab eigenvalue/eigenvector functions.

For each pair of E-vectors, you need to compute the dot products and show the output.

$$\text{Emax} \cdot \text{Emin}$$

$$\text{Emax} \cdot \text{Emid}$$

$$\text{Emin} \cdot \text{Emid}$$

(These numbers should zero or very small.)

2. Deliverables

It will be multiple pages which must be stapled in one piece. Paper clips and dog ears will earn a grade of zero. Your lab 8 assignment must be a hardcopy document, no electronic files will be accepted. If you need help, I will look at text or pdf files in email, but no word doc files. Each matrix needs a plot, a table, some code (error free and not filled with iterations). Note that the assignment is due at 3pm Friday March 20 and not the usual 5pm.

3. How the 3×3 ellipsoid case differs from the 2×2 cases

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First there are 3 eigenvectors and eigenvalues. The eigenvectors are 3 dimensional so you must use iterate3d.in instead of iterate2d.in. Convergence often takes more iterations. Finding the middle eigenvalue requires careful reading of the L-vector handout. The ϕ angle with the z-axis can be found with a dot product. The θ angle requires more calculation as θ is restricted to the xy-plane. One plots the ellipsoid with the provided function sphereplot.in in the scilab directory. The sphereplot.in expects that the eigenvectors Emax, Emin, and Emid as well as Lmax, Lmin, and Lmid have already be correctly found. See Figure 1.