Reduced Row Echelon Form

Steven Bellenot

January 16, 2008

Row Echelon Form to Reduced Row Echelon Form

For some reason our text fails to define rref (Reduced Row Echelon Form) and so we define it here. Basically rref has more zeros than ref. That is the column above a pivot must also be zero. So

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

is in ref form but not rref form as the 2 in location $a_{1,2}$ would be zero in rref. Adding -2 times row 2 and adding to row 1 turns the matrix into B which is rref:

$$B = \begin{bmatrix} 1 & 0 & 5 & -2 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Elementary row operations like the one above can always be used to convert a ref matrix to rref matrix. Our textbook uses this form in Gauss-Jordan Elimination. Note that a matrix in rref form is also in ref form. Most graphing calculators, for example the TI-83, and Scilab have rref operators which will also do the job. (That is, rref is the name of the operator that does rref.)

Reduced Row Echelon Form Definition

We give a definition of rref that is similar to the text's ref on page 2. Rule (R3) is replaced by rule (RR3) A matrix is in reduced row echelon form if it satisfies four conditions

(R1): All nonzero rows precede (that is appear above) zero rows when both types are contained in the matrix.

(R2): The first (leftmost) nonzero element of each nonzero row is unity (the number 1).

(RR3): When the first nonzero element of a row appears in column c, then all other elements in column c are zero

(R4): The first nonzero element of any nonzero row appears in a later column (further to the right) than the first nonzero element of any preceding row.

An example

We do row operations on matrix below to convert to rref. (Similar to problem 1.29)

$$\begin{bmatrix} 1 & 2 & 1 \\ -2 & -3 & 1 \\ 3 & 5 & 0 \end{bmatrix}$$

Multiple row 1 by 2 and add to row 2

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 3 & 5 & 0 \end{bmatrix}$$

Multiple row 1 by -3 and add to row 3

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 0 & -1 & -3 \end{bmatrix}$$

Multiple row 2 by 1 and add to row 3

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

We are now in ref, we continue: multiple row 2 by -2 and add to row 1

$$\begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

The matrix is now in rref.