Directions: Use only **ONE** side of each page, use ink and a staple. This lab should resonate resonance with good vibrations.

This lab is about linear approximations to non-linear second order ODEs. You will need to convert the 2nd order ODE to a 1st order system of ODEs so that scilab can solve it. (This convertion is given in section 7.1 of the ODE text.) On page 21 of the ODE text, the equation of an undamped pendulum is derived. We consider the unforced damped equation

$$y'' + y' + \sin y = 0 \qquad (A)$$

where y is the angle θ the pendulum makes with the horizontal. Since the Taylor series of

$$\sin y \approx y - \frac{y^3}{6} + \frac{y^5}{120} + \dots$$

we have a number of approimations. The most important is the linear approximation

$$y'' + y' + y = 0 \qquad (B)$$

We also consider the fifth order approximation

$$y'' + y' + y - \frac{y^3}{6} + \frac{y^5}{120} = 0 \qquad (C)$$

and briefly the third order approximation

$$y'' + y' + y - \frac{y^3}{6} = 0 \qquad (D)$$

to expose short comings that truncations can cause.

We consider the initial value problem $y(0) = 0$ and $y'(0) = v$ where v takes on the increasing sequence of values 1, 3 and 5.

Don't forget to explain how you got your numbers from scilab Also remember clarity and presentation. For scilab, or almost any other numerical solver, you must first convert the 2nd order ODE into a 1st order system by letting $z = y'$.

1. Consider the case $v = 1$, In this range the equations A, B and C closely agree. Complete the table below to show this clossness. The next equilibrium is the next time t when $y(t) = 0$, likely you will need to interpolate to find t. Also provide a plot of the three solutions.

equation	maximum (t,y(t))	next equilibrium	minimum (t,y(t))	next equilibrium
A	$(?, ?)$	$t = ?$	$(?, ?)$	$t = ?$
B	$(?, ?)$	$t = ?$	$(?, ?)$	$t = ?$
C	$(?, ?)$	$t = ?$	$(?, ?)$	$t = ?$

2. Next consider $v = 3$, in this range A and C agree well but differ somewhat from B. The next equilibrium is the next time t when $y(t) = 0$. Provide a similar plot and complete the same table

equation	maximum (t,y(t))	next equilibrium	minimum (t,y(t))	next equilibrium
A	$(?, ?)$	$t = ?$	$(?, ?)$	$t = ?$
B	$(?, ?)$	$t = ?$	$(?, ?)$	$t = ?$
C	$(?, ?)$	$t = ?$	$(?, ?)$	$t = ?$

3. All bets are off by the time $v = 5$. Explain what happens for A in particular the limiting value as $t \to \infty$. Compare to B. Provide a plot.

4. All bets are off by the time $v = 5$. Explain what happens for D in particular the limiting value as $t \to \infty$. Compare to C and provide a plot. One of the error messages given by scilab might indicate that your range on t is too large.