MAC 2311 Calculus 1 **Test 3** 21 Mar 2007 Name: ______________________________

Show **ALL** work for credit; Give **EXACT** answers when possible; **Simplify** answers;

1. Find the position $s(t)$ if the velocity is given by $v(t) = t^2 + t^{-2}$ and $s(3) = 27$.

2. Find the critical points of $f(x) = x^2 e^{-6x}$

3. Find the $\lim_{x \to \infty} x^2 e^{-6x}$

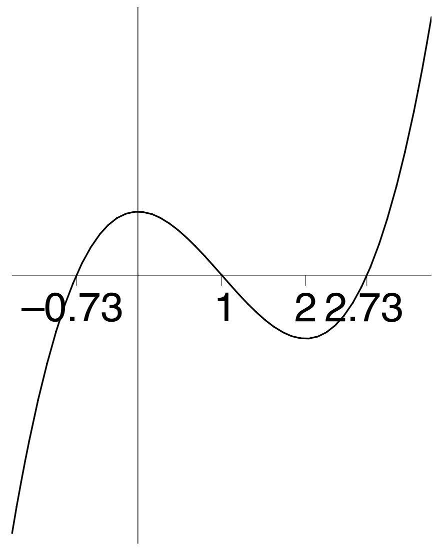
4. For $f(x) = \ln x$, $a = 1$ and $b = e$, find all c that satisfies both conclusions of the Mean Value Theorem, one of which is

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

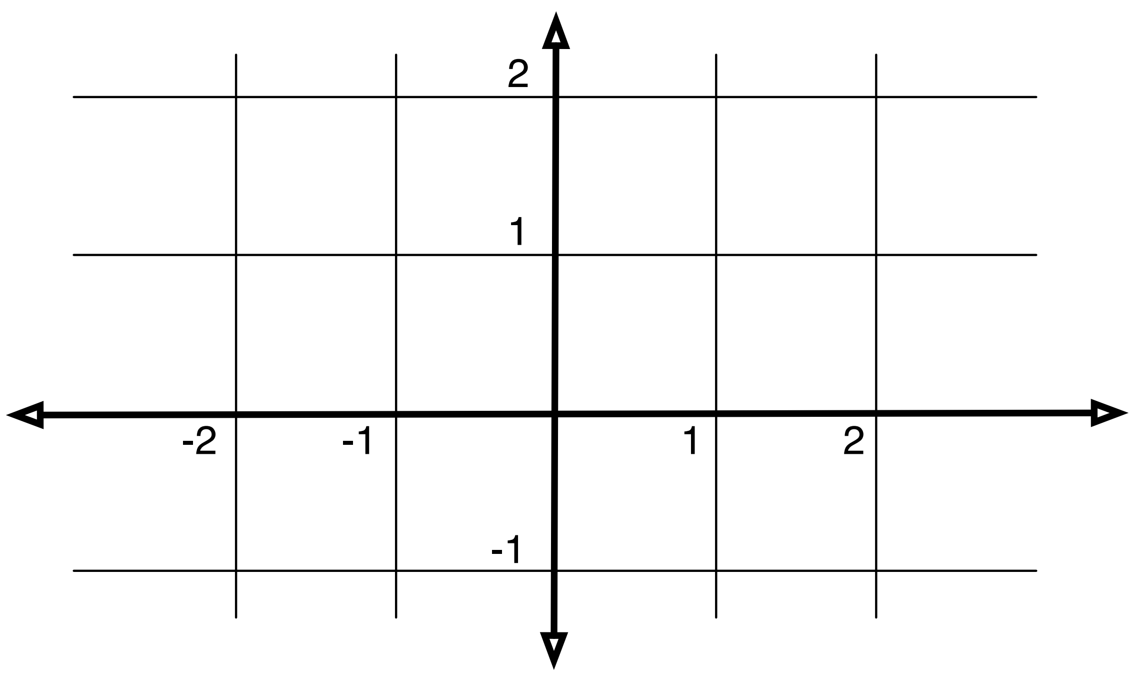
5. Find the $\lim_{x \to 0} \dfrac{\sin x^2}{\cos x - 1}$

6. Find the absolute minimum and maximum **VALUES** of $F(x) = \frac{x}{1+x^2}$ on $[-2, 3]$

7. Here is the graph of $f'(x)$, [NOT the graph of $f(x)$] find the points of inflection of f and the (open) intervals where f is smiling (concave up).



8. Draw the graph of $f(x)$ that fits the given information: $f(0) = 2$, $f'(0) = 0$; $\lim_{x\to-\infty} f(x) = -1$, $\lim_{x\to\infty} f(x) = 1$; $f'(x) > 0$ for $-\infty < x < 0$, $f'(x) < 0$ for $0 < x < \infty$; $f''(x) > 0$ for $-\infty < x < -1$ and for $1 < x < \infty$, and $f''(x) < 0$ for $-1 < x < 1$.



9. Is the statement True or False? Give a brief reason why.

   (a) If $f'(c) = 0$ and $f''(c) > 0$ then $f(x)$ as a local max at $x = c$

   (b) The second derivative test fails for $f(x) = x^{100}$ at $x = 0$

   (c) If $f'(c)$ does not exist, then $x = c$ is a critical point of $f(x)$

   (d) If $f''(c) = 0$, then $x = c$ is a point of inflection for $f(x)$.

   (e) If $f(x) > 0$, $\lim_{x\to\infty} f(x) = 0$, and $f'(x) < 0$ then $f''(x) < 0$

10. Find the **MAXIMAL AREA** that a rectangle inscribed into a semi-circle of radius R can have.