Geodesics: Gravity is Geometry and not a Force

Oh leave the Wise our measures to collate
One thing at least is certain, light has weight
One thing is certain and the rest debate
Light rays, when near the Sun, do not go straight
Sir Arthur Eddington,
in parody of the *Rubaiyat* of Omar Khayyam

The year was 1919, the location was Principle Island in West Africa, the event was an solar eclipse and Arthur Eddington was attempting to test the general theory of relativity. A pattern of stars called the Hyades were photographed next to the sun as they appeared at the moment of the total eclipse. In contrast to other photos, this photo showed tiny displacement towards the sun, evidence that light was bent by the sun's gravity. Newton's force of gravity could not explain light bending. Light is massless, and Netown's gravitational force is proportional to the product of the masses. (But Newton had predicted that light would bend near a massive body.) Einstein's theory predicted that light would travel along geodesics in a universe whose geometry was shaped by mass. It is an instant in time when the world changed.

In 1979, it was realized that what at first looked like twin quasars were in fact the same quasar behind a massive gravity well. Light from the quasar was bent along more than one path, and so was visible as if quasar was in more than one location in the night sky. This effect is called Gravitational Lensing. The final new piece to the project will be to simulate a gravitational lensing by a "black hole" shaped surface. But first an attempt at "light" humor.

> Perhaps one of the more famous lines from the 1977 movie Star Wars, is when Obi-Wan Kenobi says "Use the Force Luke". Now you know that there is no force, only geometry. Unfortunately "Use the Geometry Luke" confused the test audiences and so the line was changed to please the Newtonians amongsth us. You might also be amused by a google search on "use the source, luke".

As mentioned above, the final new piece to the project will be to simulate a gravitational lensing by a "black hole" shaped surface. We will do this by modifying the maple worksheet you used in the last part of the project. We will use the surface $z = f(x, y) = -e^{-(x^2+y^2)}$ to represent the curved space, the quasar will be located at $Q(-5, 0.1, -e^{-25.01})$ and the earth will be at $E(4, -0.05, -e^{-16.0025})$. You are to find two geodesics going from Q to E and print out both the Maple and the graph. (One geodesic on either side of the black whole. The geodesics just need to "look" right so meeting at earth is only approximately correct, but good enough to fool the eye.) Your graph needs to have a couple of maple options added, namely scaling=constained, axes=BOXED and title="Your title" where your title needs to have your groups name. Maple doesn't understand e^x until you define $e := \exp(1)$. Alternately you can use the $\exp(x)$ form in Maple. Ask me if you can't figure out how to add the options from the help pages. Please delete unnecessary output from the maple worksheet before printing.

Eventually all parts of the project will collected into a typed report, and EVENTUALLY has come! This has to to be typed. This project will be graded 80% on mathematical correctness (see the list below) and 20% on presentation and clarity. There are a small number of bonus points available for wow value.

Note that **quality of presentation is extremely important**. It is not enough merely to produce an answer: the method by which you obtain it must be sound, and you must clearly demonstrate that you understand it. Therefore, there will be penalties (commensurate with degree of infraction) for bad presentation which includes bad grammar, illegibility, incompleteness, incoherence and untidiness especially on the written assignment. Even the format of is important, use one side of the paper only, with empty margins along the left side and top of each page and with multi-page answers stapled together (do NOT use dog ears). (There is a stapler in 208 Love you can use.) Unstapled assignments will not be accepted. Late assignments will not be accepted without prior approval.

Each member of the group will submit a "group evaluation" giving their impression of the relative contribution of each member to the group's effort. These evaluations need to be typed and are due with the project. It is not guaranteed that each member of the group will receive the same grade. Note that this is not the usual evaluation form used for the other parts of the project.

Due Date: Thursday 8 April 2004

Mathematically the tasks are the following, each will receive equal value.

1. Part 1A(i) A "nice" picture and the computation of the coordinates of the two points on the unit sphere.
2. Part 1A(ii) The lengths of great circle path and 30 N path by geometry.
3. Part 2A For the given $\vec{r}(t)$, $\|\vec{r}(t)\|$, velocity, speed and acceleration.
4. Part 2B(i) Parametric equations of the 30 N curve and compuation of its arclength.
5. Part 2B(ii) Parametric equations of the great circle curve and compuation of its arclength.
6. Part 3A(i) Show 30 N curve does not satisfy the acceleration is everywhen parallel to the surface normal condition.
7. Part 3A(ii) Show any great circle does satisfy the three conditions of Fact 1.
8. Part 3B(i) Derive the "dot product rule". No special cases.
9. Part 3B(ii) Use the "dot product rule", on curves of constant speed. No special cases.
10. Part 4A(i) Example of line in xy-plane without constant speed.
11. Part 4A(ii) Use Fact 1 to show lines with the "usual form" are geodesics for the xy-plane.
12. Part 4A(iii) Show the converse, any curve $\langle x(t), y(t), z(t)\rangle$ which satisfies the three conditions of Fact 1 to be a geodesics for the xy-plane are actually lines of the form $\langle x_0 + at, y_0 + bt, 0\rangle$.
13. Part 4B(i) Show the given helix satisfies the conditons of Fact 1.
14. Part 4B(ii) Show the converse, if $\langle x(t), y(t), z(t)\rangle$ which satisfies the three conditions of Fact 1 to be a geodesics for the $x^2 + y^2 = 1$-plane is a helix of the given form.
15. Part 5A(i) If $t = t_0$ is a time when $\vec{r}(t)$ is at a minimal distance from the origin, then $\vec{r}(t_0) \perp \vec{v}(t_0)$. No special cases.
16. Part 5A(ii) Convert the given spherical equations into rectangular coordinates.
17. Part 5A(iii) Convert the rectangular coordinates equation for the $\alpha = \arcsin(1/2)$ into one that has no trig or arctrig functions and is reduced to a fraction in lowest terms.
18. Part 5A(iv) Show the last equation has constant speed.
19. Part 5B(i) Show the two curves are lines.
20. Part 5B(ii) The maple plot. Please add the title, axes=boxed.
21. Part 6 The maple plot of the gravity lens. Please add the title, axes=boxed, scaling=constrained.