Geodesics — Project Part 4

We consider two new surfaces. The first is the simplest possible surface, namely the xy-plane given by the equation $F(x, y, z) = 0$ where $F(x, y, z) = z$. We know the geodesics in this case, but we will use Fact 1 to check. The second is almost as easy, it is the (infinite) unit circular cylinder given by the equation $F(x, y, z) = 1$ where $F(x, y, z) = x^2 + y^2$. Have you seen a squirrel chase another squirrel around a pine tree? This is why they use helical paths. The reason why determining the geodesics of the cylinder is simple, is because one can easily wrap a plane around a cylinder. Draw a diagonal line on piece and wrap it around a tube to see a helix. We restate Fact 1 from Part 3 which we will continue to accept as truth without proof.

Fact 1. If $\vec{r}(t)$ has constant speed, is on the surface S and $\vec{a} = \frac{d^2\vec{r}(t)}{dt^2}$ is parallel to the normal of S at $\vec{r}(t)$ for each t, then $\vec{r}(t)$ is a geodesic for S, and conversely.

The goal for this assignment is to use Fact 1 to characterize the geodesics of the plane and the circular cylinder $x^2 + y^2 = 1$.

Part A. (i) Show there are parametric equations of lines in xy-plane without constant speed. (ii) Show that the usual parametric equations of any line in the xy-plane is a geodesic by checking Fact1. (iii) Show the converse, if $\langle x(t), y(t), z(t)\rangle$ is a geodesic in the xy-plane, than it has the form $\langle x_0 + v_1 t, y_0 + v_2 t, z_0 + v_3 t\rangle$ for some constants x_0, v_1, y_0, v_2, z_0 and v_3.

Part B. (i) Show for any constants a and b that $\vec{r}(t) = \langle \cos(at), \sin(at), bt\rangle$ is a geodesic for the surface $x^2 + y^2 = 1$. (ii) Show the converse, that any geodesic in the surface $x^2 + y^2 = 1$ has this form for some constants a and b.

Hints: Fact 1 says to show $x = x(t), y = y(t), z = z(t)$ is a geodesic you must check three things: (a) The curve is on the surface, that is $F(x(t), y(t), z(t))$ is the correct constant; (b) The curve has constant speed, that is $\sqrt{(x')^2 + (y')^2 + (z')^2}$ is constant; and (c) The acceleration $\langle x''(t), y''(t), z''(t)\rangle$ is parallel to the normal $\nabla F(x(t), y(t), z(t))$.

Make sure you know what converse means and what 'and conversely' means. To show the converse, you will have to solve some very simple differential equations of the form $f'' = 0$ (H6.3 H11.1).

A. (i) Accelerate as you go straight, (iii) The (a) condition will determine $z(t)$ while the normal condition (c) will determine the other two.

B. For the converse use the (c) condition to show $z = bt$ and then use the constant speed condition (b) and the surface to show that the $x(t)$ and $y(t)$ must be traveling around the unit circle at constant speed.

Eventually all parts of the project will collected into a typed report, but this assignment does not have to be typed.

Note that **quality of presentation is extremely important**. It is not enough merely to produce an answer: the method by which you obtain it must be sound, and you must clearly demonstrate that you understand it. Therefore, there will be penalties (commensurate with degree of infraction) for bad presentation which includes bad grammar, illegibility, incompleteness, incoherence and untidiness especially on the written assignment. Even the format of is important, use one side of the paper only, with empty margins along the left side and top of each page and with multi-page answers stapled together (do NOT use dog ears). (There is a stapler in 208 Love you can use.) Unstapled assignments will not be accepted. Late assignments will not be accepted without prior approval.

Finally group projects require each member fill out a separate sheet which evaluates the relative contributions of the other members of the group. This form is due at the same time as the project.