Geodesics — Project Part 3

How can we tell if a parameteric path $\vec{r}(t)$ with constant speed is a geodesic in a given surface S? Obviously if $\vec{r}(t)$ must be in S for all time t and hence it must bend as S bend. Intuitively, it must bend for no other reason, all of $\vec{r}$ bends are dictated by the surface S. Since $\vec{r}$ has constant speed, this means the acceleration is all due to the shape of the path. Further it will turn out that the acceleration is normal to the velocity in this case. Hence it is not unreasonable to think that $\vec{r}$ is a geodesic if its acceleration is parallel to the normal of S at each point. We state this as Fact 1 which we will accept as truth without proof.

Fact 1. If $\vec{r}(t)$ has constant speed, is on the surface S and $\vec{a} = \frac{d^2\vec{r}(t)}{dt^2}$ is parallel to the normal of S at $\vec{r}(t)$ for each t, then $\vec{r}(t)$ is a geodesic for S, and conversely.

The goal for this assignment is to use Fact 1 to show a couple of things for our sphere of radius one. First that the path from point A at 30N 0W to point B at 30N 90E along the latitude 30N is not a geodesic for the sphere. Second that any great circle path is a geodesic. Finally we will derive the formulas needed to show that in curves with constant speed the velocity is perpendicular to the acceleration.

Part A. Show the parametric equation from Part 2 for the path from point A at 30N 0W to point B at 30N 90E along the latitude 30N is not a geodesic of the sphere of radius 1. But that any great circle is a geodesic.

Part B. Prove that for curves with constant speed, the velocity is perpendicular to the acceleration by deriving the dot product rule formula (where $\vec{p} = \vec{p}(t)$ and $\vec{r} = \vec{r}(t)$ are both functions of t)

$$\frac{d}{dt}(\vec{p} \cdot \vec{r}) = \frac{d\vec{p}}{dt} \cdot \vec{r} + \vec{p} \cdot \frac{d\vec{r}}{dt}$$

and applying it to $\vec{v}(t) \cdot \vec{v}(t) = \|\vec{v}(t)\|^2 = \text{constant}$ to show $\vec{v}(t) \cdot \vec{a}(t) = 0$.

Hints: A. Show the normal at the point P of the unit sphere is $\overrightarrow{OP}$ via the gradient.
B. Use lots of subscripts on the dot product rule.

Eventually all parts of the project will collected into a typed report, but this assignment does not have to be typed.

Note that **quality of presentation is extremely important**. It is not enough merely to produce an answer: the method by which you obtain it must be sound, and you must clearly demonstrate that you understand it. Therefore, there will be penalties (commensurate with degree of infraction) for bad presentation which includes bad grammar, illegibility, incompleteness, incoherence and untidiness especially on the written assignment. Even the format of is important, use one side of the paper only, with empty margins along the left side and top of each page and with multi-page answers stapled together (do NOT use dog ears). (There is a stapler in 208 Love you can use.) Unstapled assignments will not be accepted. Late assignments will not be accepted without prior approval.

Finally group projects require each member fill out a separate sheet which evaluates the relative contributions of the other members of the group. This form is due at the same time as the project.