Geodesics — Project Part 2

Basically we are going to repeat Part 1 except we will use calculus instead of geometry to compute the lengths. We want to use the arclength formula [page 796 of the text]. There are two steps we need to do, first we need to find parametric equations and second we need to do the integral. This is not hard for the path along the latitude 30N, but gets messy for the great circle. Perhaps paradoxically, it is easier to do the great circle case in abstract generality first.

> *The goal for this assignment is to use the vector calculus methods of chapter 17 to compute the distances from point A at 30N 0W to point B at 30N 90E along two paths on a sphere of radius one. The first path is along the latitude 30N. The second path is along the great circle through these two points. We need to find a vector parametric formula $\vec{r}(t)$ for t in $[a,b]$ and then compute the length with the arclength formula $\int_a^b \|\frac{d\vec{r}(t)}{dt}\|\, dt$ for each path.*

Part A. Suppose $\vec{u} = \langle u_1, u_2, u_3\rangle$ and $\vec{v} = \langle v_1, v_2, v_3\rangle$ are perpendicular unit vectors so that $\vec{u}\cdot\vec{v} = 0$ and $\vec{u}\cdot\vec{u} = \vec{v}\cdot\vec{v} = 1$. Consider the parametric equation $\vec{r}(t) = R\vec{u}\cos t + R\vec{v}\sin t$ which is a circle in the plane determined by the origin O and the points $U(u_1,u_2,u_3)$ and $V(v_1,v_2,v_3)$. Show $\|\vec{r}(t)\| = R$ and compute the velocity, speed and acceleration of $\vec{r}(t)$. Your answers should make best use of vector notation. (One could define the curve $\vec{r}$ as $x = Ru_1\cos t + Rv_1\sin t$, $y = Ru_2\cos t + Rv_2\sin t$, and $z = Ru_3\cos t + Rv_3\sin t$ but that wouldn't be the best use of vector notation as there are subscripts showing.)

Part B. Find a and b and a vector parametric formula $\vec{r}(t)$ for t in $[a,b]$ and use it to compute the length with the arclength formula $\int_a^b \|\frac{d\vec{r}(t)}{dt}\|\, dt$ for each path. Note that if the vector starts at the origin O the head of $\vec{r}(a)$ is the point A and the head of $\vec{r}(b)$ is the point B. There is a further requirement for $\vec{r}$, namely that it have constant speed. (It will not have constant velocity.) Constant speed is needed for the next part of the project, it makes the arclength integrals extremely easy to evaluate, and all the obvious parametric forms have this property, but it is not automatic.

Hints: A. Use the dot product to compute norms $\|\alpha\vec{u} + \beta\vec{v}\|^2 = (\alpha\vec{u} + \beta\vec{v})\cdot(\alpha\vec{u} + \beta\vec{v})$ and distribute to get $\alpha^2 + \beta^2$.
B. For the great circle path use Part A with $R = 1$. There are more than one possible choice for $\vec{u}$ and $\vec{v}$ here is one suggestion. Let the unit $\vec{u}$ be $\overrightarrow{OA}$, and write $\overrightarrow{OB}$ as a sum of two vectors one parallel and one perpendicular to $\vec{u}$.

Eventually all parts of the project will collected into a typed report, but this assignment does not have to be typed.

Note that **quality of presentation is extremely important**. It is not enough merely to produce an answer: the method by which you obtain it must be sound, and you must clearly demonstrate that you understand it. Therefore, there will be penalties (commensurate with degree of infraction) for bad presentation which includes bad grammar, illegibility, incompleteness, incoherence and untidiness especially on the written assignment. Even the format of is important, use one side of the paper only, with empty margins along the left side and top of each page and with multi-page answers stapled together (do NOT use dog ears). (There is a stapler in 208 Love you can use.) Unstapled assignments will not be accepted. Late assignments will not be accepted without prior approval.

Finally group projects require each member fill out a separate sheet which evaluates the relative contributions of the other members of the group. This form is due at the same time as the project.