Geodesics — Project Part 1

The topic for this semesters project is **geodesics** on surfaces in 3-space. The general theory of geodesics is quite complex, but it turns out that one can illustrate the general theory quite well with the tools of calculus 3.

A formal definition of a geodesic will be given later in the semester, but for now we will use the intuitive idea that a geodesic is a shortest path. The easiest example surface is the plane, and geodesics in the plane are straight lines. The second easiest surface is the sphere and that is the topic for this assignment.

Geodesics on the sphere are great circles. A great circle is formed when a plane containing the center of the sphere is intersected with the sphere. The equator and longitude circles (which contain both the north and south poles) are examples of great circles. Latitude circles other than the equator are not great circles. For example, both Tallahassee and New Orleans are roughly 30 degrees north latitude (actually the airports are at 30.38N 84.37W and 30.03N 90.03W respectively), but the shortest path between them is not following the line of latitude, but is actually a path that has a non-trivial northern component.

> *Your job for this assignment is to compute the distances from 30N 0W to 30N 90E along two paths on a sphere of radius one. The first path is along the latitude 30N. The second path is along the great circle through these two points.*

Before listing the requirements let's observe a weakness in the intuitive definition of a geodesic as a shortest path. Suppose you are in Tallahassee and you want to go to the north pole. The geodesic goes straight north. If you continue going straight after reaching the north pole, you will be going due south towards the south pole. You would have followed a geodesic from Tallahassee to the south pole, but you have not taken the shortest path. The shortest path would, of course, be due south from Tallahassee to the south pole. That is a geodesic is the shortest path "for a while" but perhaps not "forever" like it is in the plane.

Requirements: Your solution must use geometry and not the calculus of chapter 17. The vector techniques are ok. Your answers must be exact. (This is not a physics course where the approximate 1.414 is prefered over the exact $\sqrt{2}$.) The quality of presentation is extremely important (see below). Eventually all parts of the project will collected into a typed report, but this assignment does not have to be typed.

Hints: 30N 0W is on the xz-plane while 30N 90E is on the yz-plane. From plane geometry, $s = r\theta$ is the formula for arclength of a sector of a circle of radius r with central angle θ in radians.

Note that **quality of presentation is extremely important**. It is not enough merely to produce an answer: the method by which you obtain it must be sound, and you must clearly demonstrate that you understand it. Therefore, there will be penalties (commensurate with degree of infraction) for bad presentation which includes bad grammar, illegibility, incompleteness, incoherence and untidiness especially on the written assignment. Even the format of is important, use one side of the paper only, with empty margins along the left side and top of each page and with multi-page answers stapled together (do NOT use dog ears). (There is a stapler in 208 Love you can use.) Unstapled assignments will not be accepted. Late assignments will not be accepted without prior approval.

Finally group projects require each member fill out a separate sheet which evaluates the relative contributions of the other members of the group. This form is due at the same time as the project.