Extra Answers

- **contours for 12.2#7** The contours are circles centered at the orgin, if $z = c$, a constant, then the radius of the circle is $\sqrt{5-c}$. Usually the contours are done for "equally spaced" z values, so the the contours themselves are not equally spaced, but get closer and closer as we get farther from the origin.

- **Prob 10 test1 spring 2000** A-VII; B-IV; C-II; D-III; E-I; F-VI; G-VIII; H-V.

- **Prob 10 test2 spring 1996** Contour 1 is $\sin(x+y)$; contour 2 is $\sin(xy)$; contour 3 is $\sin x + \sin y$; and contour 4 is $\sin x \sin y$

- **POLAR** Change to polar $x = 3$ is $r\cos\theta = 3$ or $r = 3\sec\theta$; $x^2 + y^2 = 9$ is $r = 3$; $x = -y^2$ is $r = -\cot\theta\csc\theta$; $x + y = 9$ is $r = 9/(\cos\theta + \sin\theta)$; $x^2+y^2 = 2cx$ is $r = 2c\cos\theta$; $x^2-y^2 = 1$ is $r^2 = 1/(\cos^2\theta - sin^2\theta) = \sec 2\theta$ Change to Cartesian $r = 2$ is $x^2 + y^2 = 4$; $r\cos\theta = 1$ is $x = 1$; $r = 3\sin\theta$ is $x^2+y^2 = 3y$ (a circle); $r = 2\sin\theta + 2\cos\theta$ is $x^2+y^2 = 2y+2x$ (another circle); $r = \csc\theta$ is $y = 1$; $r = \tan\theta\sec\theta$ is $x^2 = y$ (a parabola).

- **LIMIT** Do the following have limits? (Hint: convert to polar.)

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2} \qquad \lim_{(x,y)\to(0,0)} \frac{x^3-y^3}{x^2+y^2}$$

$$\frac{x^2-y^2}{x^2+y^2} = \frac{r^2\cos^2\theta - r^2\sin^2\theta}{r^2} = \cos 2\theta$$

This has no limit (along x axis and along $x = y$ give different values).

$$\frac{x^3-y^3}{x^2+y^2} = \frac{r^3\cos^2\theta - r^3\sin^2\theta}{r^2} = r\cos 2\theta \to 0 \qquad \text{as} \quad r \to 0$$

- **14.6#10**

$$\frac{\partial z}{\partial u} = (e^y)(\frac{1}{u}) + (e^y + xe^y + ye^y)(0)$$
$$\frac{\partial z}{\partial v} = (e^y)(0) + (e^y + xe^y + ye^y)(1)$$

- **14.6#14** I would always use arctan and not $\tan^{-1}$.

$$\frac{\partial z}{\partial u} = (\frac{1}{1+\frac{x^2}{y^2}})(\frac{1}{y})(2u) + (\frac{1}{1+\frac{x^2}{y^2}})(-\frac{x}{y^2})(2u)$$

$$\frac{\partial z}{\partial v} = (\frac{1}{1+\frac{x^2}{y^2}})(\frac{1}{y})(2v) + (\frac{1}{1+\frac{x^2}{y^2}})(-\frac{x}{y^2})(-2v)$$

- **SKEW LINES** Find the minimum of $f(s,t) =$ the distance-squared between the points on the two lines $x = 1; y = 1; z = t$ and $x = 3 + s; y = 0; z = -s$ using chapt 15 techniques, also find the points on the two lines nearest each other. Answer, the points are $(1,1,2)$ and $(1,0,2)$ and the distance is one.

- **LAGRANGE** Find the maximum and minimum values of $f(x,y,z) = x+2y+3z$ subject to the two constraints $x^2+y^2+z^2 = 1$ and $x+y+z = 1$. Answer Max is $2 + \frac{2}{3}\sqrt{3}$, min is $2 - \frac{2}{3}\sqrt{3}$.

- **TRIPLE INTEGRAL** Change the order of integration of

$$\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f\,dz\,dy\,dx$$

in the orders $dx\,dy\,dz$ and $dy\,dz\,dx$. See region2.mws in the maple subdirectory.

$$\int_0^1 \int_0^{1-z} \int_0^{y^2} f\,dx\,dy\,dz \qquad \int_0^1 \int_0^{1-\sqrt{x}} \int_{\sqrt{x}}^{1-z} f\,dy\,dz\,dx$$

- **EULER**. Consider the vector field $\vec{F} = \langle -y - x/10, x - y/10 \rangle$
A. Show $\vec{r}(t) = \left\langle e^{-t/10}\cos t, e^{-t/10}\sin t \right\rangle$ is a flow for $\vec{F}$
B. Use Euler's method to approximate the flow which starts at $(1,0)$ by completing a table that starts like the one below with as much accuracy has your TI-89 can give. [Check to see that you are in both radian mode and using the Euler method]. Do five steps of size $\Delta t = 0.1$
Answers A. Compute $x'(t)$ and $y'(t)$ and plug them into the equation.
B. $\Delta t = 0.1$ so we have

t	x	y	$x'(t)$	$y'(t)$	Δx	Δy
0	1	0	-0.1	1	-0.01	0.1
0.1	0.99	0.1	-0.199	0.98	-0.0199	0.099
0.2	0.9701	0.199	?	?	?	?
0.3	?	?	?	?	?	?
0.4	?	?	?	?	?	?
0.5	?	?	?	?	?	?

- **18.4#20** πab

- **19.3#6** $\pi\sqrt{6}$