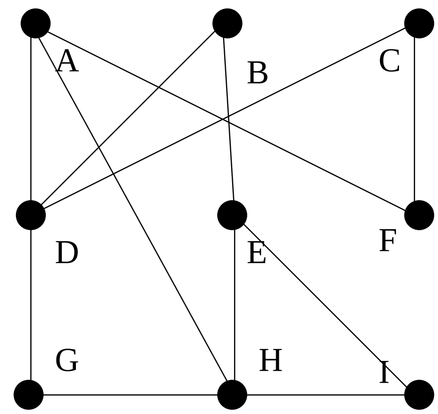
MAD 3104 Discrete Math 1 **Test 2** 25 Oct 1995

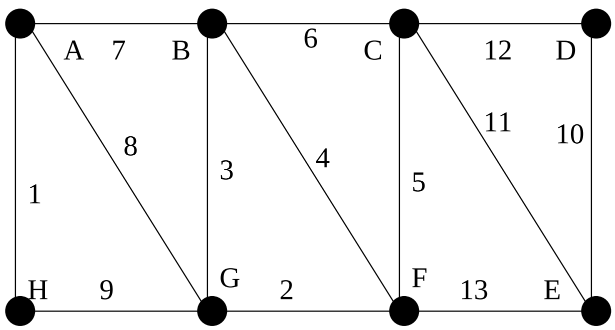
Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper.

1. Two unrelated short problems.
 A. How many ways can a 333-element subset be selected from a set with 666 elements?
 B. Make a pairwise nonisomorphic list of all 6 trees with 5 edges.

2. For the graph to the right.
 A. Find the DFS spanning tree.
 B. Find the BFS spanning tree.

3. For the graph to the right
 A. Find a cycle.
 B. Find a circuit that is not a cycle.
 C. Find an Euler path.

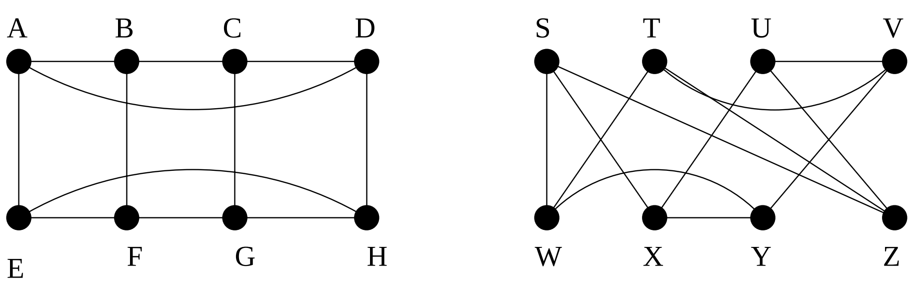




4. For weighted graph above, list the edges IN THE ORDER SELECTED by:
 A. Prim's Algorithm (starting at A).
 B. Kruskal's Algorithm.

5. For the weighted graph above problem #4 apply Dijkstra's Shortest Path Algorithm starting at vertex A. Show the RESULTING labels for each vertex, show the shortest PATH obtained from A to E and STATE its length. (Be sure to answer ALL parts of this question!)

6. Decide if the two graphs below are isomorphic or not. If they are isomorphic, then give an isomorphism. If they are not isomorphic, then explain why they are not isomorphic.



7. Prove by induction (on n) $1 \cdot 2 + 2 \cdot 3 + \cdots + n \cdot (n+1) = \frac{n(n+1)(n+2)}{3}$.

8. Prove by contradiction: Prove a graph with 61 edges and 30 vertices has a vertex of degree at least 5.

9. Given $a_0 = 0$, $a_1 = 3$ and $a_{n+1} = 2a_n + 7a_{n-1}$ for $n \geq 1$. Prove by induction, for each integer $n \geq 0$, $a_n < 4^n$.

10. Prove by induction: For each integer $n \geq 0$, $\frac{(2n)!}{n!2^n}$ is an integer. [By definition $0! = 1$.]