

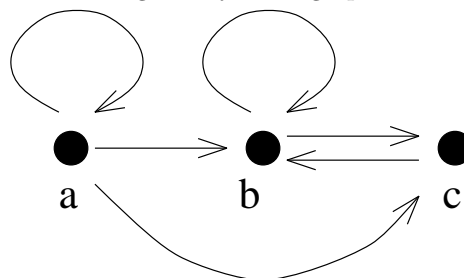
Show **ALL** work for credit; be neat; and use only **ONE** side of each page of paper.

1. Use De Morgan's and/or the distributive laws to simplify:

- A.  $\overline{A} \cap (A \cup B)$   
 B.  $\overline{(A - B)} \cap A$

2. Give counterexamples to the statements below. The relation  $R$  is the one given by the digraph below.

- A.  $R$  is reflexive.  
 C.  $R$  is symmetric.  
 D.  $R$  is anti-symmetric.  
 E.  $R$  is transitive.



3. Solve for  $m$  in  $Z_{13}$ . Write your answer so that  $0 \leq m < 13$ .

- A.  $[5] + [12] = [m]$   
 B.  $[5][9] = [m]$   
 C.  $[5][m] = [3]$   
 D.  $[7]^{101} = [m]$

4. Construct a truth table for  $(p \wedge q) \rightarrow (\sim p \vee q)$ .

5. For each part, decide whether the logic is valid or invalid and draw a Venn diagram to support your answer.

- A. If  $x + 2 = x$ , then  $x$  is blue.  $x + 2 = x$ . Therefore,  $x$  is blue.  
 B. If  $T$  is a rectangle, then  $T$  is a square.  $T$  is a square. Therefore,  $T$  is a rectangle.

6. Negate the following statements and re-write them so that words like "not" or "no" are not used.

- A. For all triangles  $T$ , the area( $T$ )  $\geq$  perimeter( $T$ ).  
 B. For some integers  $x$ ,  $x$  is odd and  $x^2$  is even.

7. Draw the digraph for the relation  $R$  on the set  $A = \{2, 3, 4, 5, 6\}$  where the relation  $R$  is defined by  $aRb \iff a = 3$  or  $b = 5$ .

8. Equivalent classes. For the given set  $A$ , the relation  $R$  is an equivalence relation, describe the the equivalence class  $[p]$ , for the given  $p$ .

- A.  $A$  is the set of reals,  $aRb \iff a^2 = b^2$ , and  $p = 4$ .  
 B.  $A$  is the points in the plane,  $(a, b)R(c, d) \iff a^2 + b^2 = c^2 + d^2$ , and  $p = (3, 4)$ .  
 C.  $A = \{1, 2, 3, 4, 5, 6\}$ ,  $aRb \iff a = b$  or both  $a \geq 3$  and  $b \geq 3$ , and  $p = 4$ .

9.  $A$  is the set of reals and  $aRb \iff a + 10 \leq b$ .

- A. Give counter-examples to show  $R$  is not reflexive and not symmetric.  
 B. Give proofs to show  $R$  is anti-symmetric and transitive.

10. Proof or disprove. Let  $A$  be the set of points in the plane and let  $R$  be the relation defined by  $(a, b)R(c, d) \iff |a - c| \leq |b - d|$ .

- A.  $R$  is reflexive.  
 B.  $R$  is symmetric.  
 C.  $R$  is anti-symmetric.  
 D.  $R$  is transitive.

Keep this sheet. Graded tests will be ready 1:30 Friday 22 Sep in 002B LOV.