## Relations

Problems: For the given A and R and each of the properties: A. reflexive, B. symmetric, C. anti-symmetric and D. transitive, decide if R has the property. If it has the property then prove it has that property or if it doesn't have the property then give a counterexample to show the property fails. (I.e. Prove or disprove.)

- 1. A is the set of real numbers and  $aRb \iff a \leq b$ .
- 2. A is the set of real numbers and  $aRb \iff a < b$ .
- 3. A is the set of real numbers and  $aRb \iff 0 \le a b \le 2$ .
- 4. A is the set of real numbers and  $aRb \iff |a-b| < 2$ .
- 5. A is the set of odd positive integers and  $aRb \iff a \neq b$  and a evenly divides b.
- 6. A is the set of real numbers and  $aRb \iff a^2 b^2 = 0$ .
- 7. A is the set of positive integers and  $aRb \iff a$  divides b.
- 8. A is the set of integers and  $aRb \iff a-b$  is odd.
- 9. A is the set of positive integers and  $aRb \iff a \equiv 1 \mod b$ .
- 10. A is the set of integers and  $aRb \iff a \cdot b$  is even.
- 11. A is the set of points in the plane and  $(a, b)R(c, d) \iff (a c)^2 + (b d)^2 \le 5$ .
- 12. A is the set of points in the plane and  $(a, b)R(c, d) \iff a + b = c + d$ .
- 13. A is the set of points in the plane and  $(a, b)R(c, d) \iff |a b| = |c d|$ .
- 14. A is the set of points in the plane and  $(a, b)R(c, d) \iff a = c$ .
- 15. A is the set of points in the plane and  $(a, b)R(c, d) \iff a = d$ .
- 16. A is the set of triangles in the plane and  $tRs \iff$  triangle t has the same area as triangle s.
- 17. A is the set of triangles in the plane and  $tRs \iff$  triangle t is similar to triangle s.
- 18. A is the set of triangles in the plane and  $tRs \iff$  triangle t has either at least as much area as triangle s, or triangle t has at least as large perimeter as triangle s.
- 19. A is the set  $\{1, 2, 3, \{1\}, \{1, 3\}, \{2\}\}$  and  $aRb \iff a \in b$ .
- 20. A is the set power set of  $\{1, 2, 3\}$  and  $aRb \iff a \subseteq b$ .