Proofs: Contradiction and Induction

1. Prove by contradiction: A graph with 100 edges and 19 vertices has a vertex of degree at least 11.
2. Prove by contradiction:
   A. Prove a graph with 35 edges and 16 vertices has a vertex of degree at least 5.
   B. Prove a graph with 45 edges and 24 vertices has a vertex of degree at most 3.
3. Prove by contradiction:
   A. Prove a graph with 40 edges and 25 vertices has a vertex of degree at least 4.
   B. Prove a graph with 50 edges and 20 vertices has a vertex of degree at most 5.
4. Prove by contradiction:
   A. Prove a graph with 41 edges and 20 vertices has a vertex of degree at least 5.
   B. Prove a graph with 49 edges and 25 vertices has a vertex of degree at most 3.

5. Prove by induction (on n) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
6. Prove by induction that $\sum_{i=1}^{n}(2i-1) = n^2$.
7. Prove by induction that $1+3+5+\cdots+(2n-1) = n^2$.
8. Prove by induction: For each integer $n \geq 0, 4^n > n^2$.
9. Prove by induction that $n! > 2^n$ for $n \geq 4$.
10. Given $a_0 = 2$, $a_1 = 0$ and $a_n = 5a_{n-1} - 6a_{n-2}$ for $n \geq 2$, prove by induction, for each integer $n \geq 0$, $a_n = 6 \cdot 2^n - 4 \cdot 3^n$.
11. Given $a_0 = 3$, $a_1 = 0$ and $a_{n+1} = 6a_n - 8a_{n-1}$ for $n \geq 1$, prove by induction, for each integer $n \geq 0$, $a_n = 6 \cdot 2^n - 3 \cdot 4^n$.