1. True or False and a brief reason why or why not. Let A, B, C and D be the matrices given below.

$$A = \begin{bmatrix} 0 & 0 & 7 \\ 13 & 0 & 0 \\ 0 & -3 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

(a) The matrix equation $AX = \begin{bmatrix} 21 \\ 65 \\ 33 \end{bmatrix}$ has $X = \begin{bmatrix} 5 \\ -11 \\ 3 \end{bmatrix}$ as a solution

(b) Using scilab notation on matrix B, $5 * B(2,:) - 3 * B(1,:)$ is $\begin{bmatrix} -3 & 5 & 2 \end{bmatrix}$

(c) The matrix equation $BX = 0$ has ∞-many solutions

(d) The determinate of matrix C is 1.

(e) The inverse of A is $\begin{bmatrix} 0 & 1/13 & 0 \\ 0 & 0 & -1/3 \\ 1/7 & 0 & 0 \end{bmatrix}$

(f) One can compute $B + C$ and its entry in the first row and first column is a 2.

(g) Using scilab, the command $x = 3 : 5 : 13$ will output

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x =
  3 5 7 9 11 13
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(h) If the $n \times n$ matrix M has a column of zeros then $\det(M) = 0$

(i) The matrix D can have $\det(D) = 0$

(j) If 5×5 matrix E is obtained from the 5×5 matrix F by interchanging row 3 with row 5, then $\det(E) = \det(F)$

2. True or False and a brief reason why or why not. Let A B and C be the matrices given below.

$$A = \begin{bmatrix} 0 & 0 & 3 \\ 5 & 0 & 0 \\ 0 & -11 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) The matrix equation $AX = \begin{bmatrix} 9 \\ 25 \\ 121 \end{bmatrix}$ has $X = \begin{bmatrix} 5 \\ -11 \\ 3 \end{bmatrix}$ as a solution

(b) The matrix B is in reduced row echelon form

(c) The matrix equation $CX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ has ∞-many solutions

(d) The determinate of matrix A is 165.

(e) The matrix product $B^2 = B$

(f) The row rank of matrix B is 3

(g) If 5×5 matrix E is obtained from the 5×5 matrix F by any elementary row operation, then $\det(E) = \det(F)$

(h) If $ad - bc = 1$ and $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $A^{-1} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

(i) Using scilab notation on matrix B, $5 * B(2,:) - 3 * B(1,:) = \begin{bmatrix} 5 & -3 & 2 \end{bmatrix}$

(j) If the $n \times n$ matrix M has $\det(M) = 0$ then M is invertible

3. True or False and a brief reason why or why not. Let A B and C be the matrices given below.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 6 & -3 & 3 \\ 10 & -5 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) The matrix equation $AX = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has $X = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$ as a solution

(b) The matrix B is in reduced row echelon form

(c) The matrix equation $CX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ has ∞-many solutions

(d) The determinate of matrix A is 15.

(e) The matrix product $B^2 = B + \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

(f) The row rank of matrix B is 3

(g) If the matrix C was the augmented matrix from a system of equations, then that system of equations would have ∞-many solutions.

(h) If X and Y are any 5×5 matrices, then $XY \neq YX$

(i) Using scilab notation on matrix A, $5 * A(2,:) - 3 * A(3,:) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$

(j) The transpose of matrix B, is the same as the matrix B.