

LAB8: GEOMETRY MEETS ITERATION PART 4

1. ASSIGNMENT

This lab continues to explore the connections between graphs of equations of the form

$$ax^2 + 2bxy + cy^2 = 1$$

and properties of the matrix

$$A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

discovered by iteration. We increase the dimension by one and the equation becomes

$$ax^2 + by^2 + cz^2 + 2dxy + 2exz + 2fyz = 1$$

and the matrix is

$$A = \begin{bmatrix} a & d & e \\ d & b & f \\ e & f & c \end{bmatrix}$$

You have been given a symmetric 3×3 matrix. For your matrix you will have to use Scilab to plot the quadratic surface, which is an ellipsoid, with the provided function `sphereplot.in`, which is in the usual location for scilab files. Explicitly your code should find the non-zero L-values and E-vectors by iterations, and compute the spherical coordinate angles between each E-vector and the x -axis and z -axis and put the answers in table like Figure 2. Note

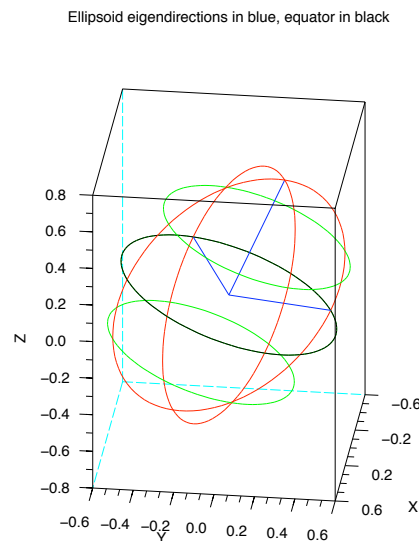


FIGURE 1. Three dimensional ellipsoid

Ellipse	L-value	E-vector	θ (degrees)	ϕ (degrees)
Max	4.1160029	(0.8627822, 0.4646697, -0.1992208)	28.305631	101.4914
Mid	2.881334	(0.5055758, -0.7929746, 0.3399770)	-57.479688	70.124524
Min	2.0300772	(2.499E-08, 0.3940474, 0.9190901)	89.999999	23.206575

FIGURE 2. Table of answers for the ellipsoid $A = [3.800413, 0.4949896, -0.2122201; 0.4949896, 3.0157437, -0.4225911; -0.2122201, -0.4225911, 2.2112574]$

that the entries in the table are not rounded off, they have as many digits as scilab gives by default. You can NOT use the built in scilab eigenvalue/eigenvector functions.

For each pair of E-vectors, you need to compute the dot products and show the output.

$$E_{\max} \cdot E_{\min}$$

$$E_{\max} \cdot E_{\text{mid}}$$

$$E_{\min} \cdot E_{\text{mid}}$$

(These numbers should zero or very small.)

2. DELIVERABLES

It will be multiple pages which must be stapled in one piece. Paper clips and dog ears will earn a grade of zero. Your lab write up must have no more than 4 pages. Your lab 8 assignment must be a hardcopy document, no electronic files will be accepted. If you need help, I will look at text or pdf files in email, but no word doc files. Each matrix needs a plot, a table, some code (error free and not filled with iterations). Note that the assignment is due at 3pm Friday October 30 and not the usual 5pm.

3. HOW THE 3×3 ELLIPSOID CASE DIFFERS FROM THE 2×2 CASES

First there are 3 eigenvectors and eigenvalues. The eigenvectors are 3 dimensional so you must use `iterate3d.in` instead of `iterate2d.in`. Convergence often takes more iterations. Finding the middle eigenvalue requires careful reading of the L-vector handout. The ϕ angle with the z -axis can be found with a dot product. The θ angle requires more calculation as θ is restricted to the xy -plane. One plots the ellipsoid with the provided function `sphereplot.in` in the scilab directory. The `sphereplot.in` expects that the eigenvectors E_{\max} , E_{\min} , and E_{mid} as well as L_{\max} , L_{\min} , and L_{mid} have already be correctly found. See Figure 1.