Functional Analysis Glossary

Steven Bellenot

August 31, 2009

- **Complex Conjugation** We use the over line notation like

  $$\overline{w+z} = \overline{w} + \overline{z}$$

  and not the star notation of our physics book by Jordan, for example

  $$(w+z)^* = w^* + z^*$$

- **Inner Product** While everyone agrees on what an *inner product* on $\mathbb{R}$, the Reals; there are two inner products on $\mathbb{C}$, the Complex field. Physics puts the conjugation on the first factor, while the second factor is more commonly used in Math books. We will do it the Physics way. We will use the $\langle \cdot, \cdot \rangle$ like the math book, the physics books uses $(\cdot, \cdot)$.

- **Manifold** Both texts use the word *manifold* for a subspace that may or may not be a closed set. This is considered a poor choice now days and it was falling in disfavor even when these books were written. The word manifold today is used only in the topological sense, i.e. the sphere is a 2-manifold. Ironically, there are Hilbert manifolds, but a non-closed subspace is not a Hilbert manifold. We will use subspace, to mean a subspace that may or may not be closed.

- **Scalars** The field of scalars is $\mathbb{K}$ which is either $\mathbb{R}$, the real number field or $\mathbb{C}$, the complex number field. Early (in the alphabet) lower case greek letters are often used for scalars:

  $$\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta, \theta, \iota, \kappa\lambda, \mu, \nu, \ldots$$

  but ϕ and ψ are usually vectors for physics.

- **Subspace** A subset S for which $S + S \subset S$ and $\mathbb{K}S \subset S$. (A subset is algebraically closed under the vector space operations.) Both texts require that a subspace is also a closed subset in the topology. We will use *closed subspace* when we want a topologically closed subspace.

- **Vectors** Lower case latin letters and ϕ and ψ often represent vectors, especially in Jordan. When the vector space is viewed as a function space, the lower case letters are also functions.